SCHOOL OF INNOVATIVE TECHNOLOGIES & ENGINEERING

Dept. of Industrial Systems Engineering

Module Information Pack

B.Eng (Hons.) Electronic Engineering Version 4.1

Optoelectronic Devices and Applications

BEE3208C

Academic Year 2025 Semester 2

Blended Mode (Online and F2F)

Programme Director: Mr. Rishi Heerasing
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Academic Tutoring: None
Lecture Day and Time: Thursdays: 11:00 – 14:00 BPML 0.02A
Credits & Level: 6 ECTS credits, Level 3
Pre-requisites (If applicable): None
Co-requisites (If applicable): None
Method of Delivery & Frequency: 45 Hours Blended Mode + 45 Hours Self-Study
Method & Criteria of Assessment: 60% Unseen Exam & 40% Coursework

LECTURE NOTES

The lecture notes are available on **Nefertum's Shrine** at https://www.rishiheerasing.net

The notes are in .pdf format so you will need Adobe Acrobat® Reader to view them.
This reader can also be downloaded from the two above-mentioned sites in the Downloads Section.

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School of Innovative Technologies and Engineering
Department of Industrial Systems Engineering
B.Eng (Hons) Electronic Engineering

Module Catalogue

Module Title: Optoelectronic Devices & Applications	**Module Code:** BEE 3208C	**Year:** 2022-2023	
Prerequisite module(s): Analogue Electronics	**Level:** 3	**Semester: 2**	**Type:** Core
ECTS Credit: 6	**ACU Credit:** 15	**Delivery Mode:** Face-to-Face	

Resource Person		**Module Coordinator**		**Programme Coordinator**	
Name:	Mr Rishi Heerasing	Name:	Mr Rishi Heerasing	Name	Mr Rishi Heerasing
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Module Description	This module investigates the operating principles and applications of various types of optoelectronic devices which play important roles in commercial and communication electronics; Fibre optic cabling and amplifier, light-emitting diodes, types of lasers and laser diodes, photo-detectors and solar cell technologies.
Module Content	Crystals and elements of crystal structure; Energy bands in semiconductors; Electron dynamics, energy levels, quantum confinement, elements of carrier transport; Optical processes in semiconductors; Optical waves: propagation in solids, waveguides; Fibre optics cable and properties; Fibre amplifier; Schottky barriers and Ohmic contacts, hetero-junctions; Light-emitting diodes; Injection lasers; Photodetectors: photo-conductors, photodiodes, avalanche photodiodes; solar cells.
Module Aims	The aims of the module are to: a. provide an advanced understanding of the physics behind optoelectronic devices b. Appreciate the design of optoelectronic devices with regards to their applications. c. Investigate and analyze the optical power ouput of laser diodes and solar cells
Learning Outcomes (LO)	At the end of the module, the learner will be able to: **LO1:** Describe key concepts in quantum and statistical mechanics relevant to physical, electrical and optoelectronic properties of materials and their applications to optoelectronic devices that emit, modulate and detect photons. **LO2:** Describe fundamental and applied aspects of optoelectronic device physics and its applications to the design and operation of laser diodes, light-emitting diodes, and photodetectors. **LO3:** Analyze optoelectronic device characteristics in detail using concepts from quantum mechanics and solid state physics. **LO4:** Investigate techniques to improve the operation of optoelectronic devices and device characteristics that have to be optimized for new applications by employing their understanding of optoelectronic device physics.

Academic Activities				
	Lecture	**Tutorial**	**Practical**	**Other (e.g. supervision and mentoring)**
Contact Hours (C)	2	1	1	1
Multiplier (M)	2	2	2	1
Number of Weeks (N)	12	12	4	6
Contact Hours/Semester (C * N)	**24**	**12**	**4**	**6**
Notional Hours/Semester (C * M * N)	**48**	**24**	**8**	**6**

Assessment Details					Examination Details	
Assessment Type	**Weightage**	**Duration (Hours)**	**Self-Study Time**	**Notional Hours**	**Paper Structure**	
Class Test 1	15%	2	12	**14**	No of Sections	N/A
Class Test 2	-	-	-	**-**	Total No of Questions	4
Assignment 1	25%	2	20	**22**	Compulsory Questions	4
Assignment 2	-	-	-	**-**	Non-Compulsory Questions	0
Mini-Project + demo	-	-	-	**-**	Paper Duration	**2.5 hrs**
Written Examination	60%	2.5	22.5	**25**		

Total Contact Hours: 24 + 12 + 4 + 6 = **46**
Total Assessment Notional Hours: 14 + 22 + 25 = **61**
Total Learning Hours: 48 + 24 + 8 + 6 + 61 = **147**
The pass mark for this module is **50%** as detailed in Section K of the B.Eng (Hons) Electronic Engineering Programme Document.

	Classification of Knowledge Areas				
	Mathematical Sciences (MS)	**Natural Sciences (NS)**	**Engineering Sciences (ES)**	**Design and Synthesis (DS)**	**Complementary Studies (CS)**
ACU Credit	-	-	15	-	-
Percentage (%)	-	-	100	-	-

Graduate Attributes *										
GA1	**GA2**	**GA3**	**GA4**	**GA5**	**GA6**	**GA7**	**GA8**	**GA9**	**GA10**	**GA11**
A	**-**	**-**	-	-	-	-	-	-	-	-

GA1: Engineering Knowledge
GA2: Problem Analysis
GA3: Design and Development of solutions
GA4: Investigations
GA5: Tool Usage
GA6: The Engineer and the World
GA7: Ethics
GA8: Individual and Collaborative Team Work
GA9: Communication
GA10: Project Management and Finance
GA11: Lifelong Learning

***Note: I – Introductory Level, D – Intermediate Development Level, A – Advanced Application Level**

Recommended Readings	Core Reading List: a) Kasap S. (2012) *Optoelectronics and Photonics: Principles and Practices*, Pearson b) Bhattacharya P. (1997) *Semiconductor Optoelectronic Devices*, Prentice-Hall Supplementary Reading List Nelson, J. (2003). The physics of solar cells. World Scientific Publishing Company

Module Map (Work Plan)

Week	**Lecture and Topics for discussion**	**Tasks / Activities**	**Learning Hours**	**Knowledge Areas (MS, NS, ES, DS, CS)**
1	Introduction to optoelectronics and optoelectronic devices	Lecture/Tutorial	3 hrs (L,T)	**ES**
2	Energy Diagrams; Carrier Concentration, Drift & Diffusion	Lecture/Tutorial	3 hrs (L,T)	**ES**
3	LED	Lecture/Tutorial	3 hrs (L,T)	**ES**
4	LED	Practical & Supervision	3 hrs (P,S)	**ES**
5	LASER	Lecture/Tutorial	3 hrs (L,T)	**ES**
6	LASER	Practical & Supervision	3 hrs (P,S)	**ES**
7	Fibre Optics and FSO	Lecture/Tutorial	3 hrs (L,T)	**ES**
8	Fibre Optics	Practical & Supervision	3 hrs (P,S)	**ES**
9	Photo-detectors and Photodiodes	Lecture/Tutorial	3 hrs (L,T)	**ES**
10	Photo-detectors and Photodiodes	Lecture/Tutorial	3 hrs (L,T)	**ES**
11	Class Test	Tutorial & Mentoring	1 hr Tutorial 2 hours class test and mentoring each	**ES**
12	Solar Cells	Lecture/Tutorial	3 hrs (L,T)	**ES**
13	Solar Cells	Lecture/Tutorial	3 hrs (L,T)	**ES**
14	Supervision for Assignment	Practical & Supervision	3 hrs (P,S)	**ES**
15	Discussion and Feedback on Performance	**Assignment Submission (25%)**	3 hrs (L.T)	**ES**

	Prepared by Examiner	**Verified by either Module Coordinator or Programme Director or Head of Department**
Name	**Rishi Heerasing**	**Sandhya Armoogum**
Signature		
Date	**9th September 2022**	**12th September 2022**

End of Module Catalogue

Slide Set 0

- Wavelength Utilization
- Growth of Internet Traffic
- Link Capacity Improvements
- Technological Trends
- Optoelectronics

What determines the wavelengths utilized?

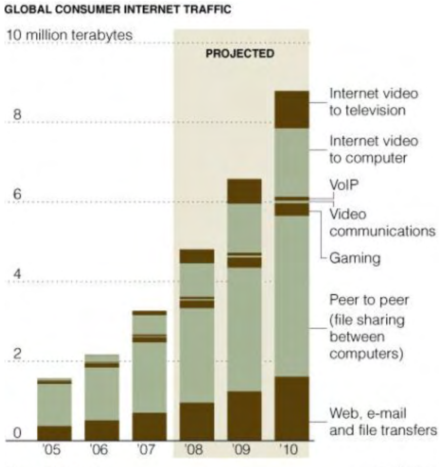
1) Human eye response (indicators and displays)

2) Transmission media
    - Fibers
    - Atmospheric Transmission

3) Source of radiation (detectors)
    - Sun (Solar Spectrum)
    - Black body – T
    - Lasers

4) Technologically available materials

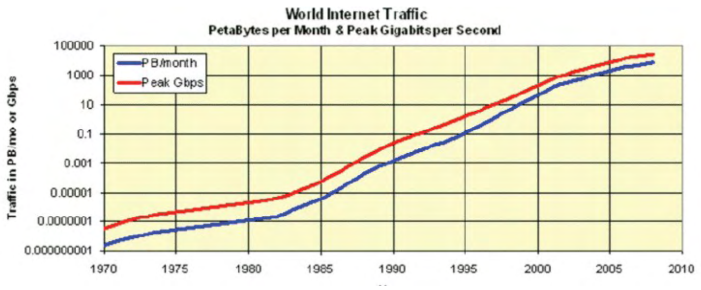
Growth of Internet Traffic



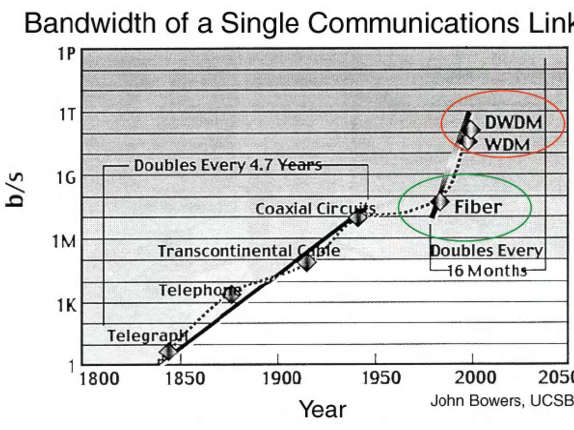
Source: Cisco Systems

THE NEW YORK TIMES

Growth of Internet Traffic



Link Capacity Improvements



Key to WDM was Er-doped fiber amplifier

Key to fiber was low loss fiber and heterojunction lasers

John Bowers, UCSB

SONET/SDH Specifications

Beginner	Intermediate	Advanced	Extreme
●	🟦	◆	🟥
OC-48 2.5 Gb/s	OC-192 10 Gb/s	OC-768 40 Gb/s	OC-3072 160 Gb/s

OC – Optical Carrier rate, OC-1 = 51.84 Mbps

Technological Trends

Computer performance continues to follow Moore's Law
- Doubling of computing power every 18 months
- Device research indicates exponential growth until 2020

Network capacity is also increasing exponentially
- Gilder's Law – "Communication capacity triples every 12 months"
- Growth at a rate greater than Moore's law requires new integrated, lower cost technologies

Required network capacity
- Desktop bandwidth needs limited only by the human eye (~2Gbps)
- Internet traffic is non local
- Number of users and hosts is growing exponentially

∴ Need long wavelength, high speed, low cost, vertical-cavity surface-emitting lasers (VCSELs) and integrated photonic ICs with higher functionality

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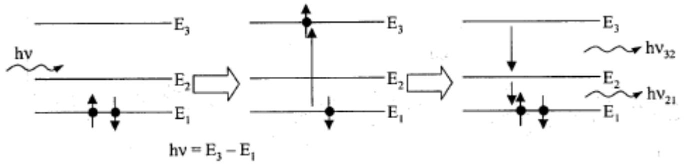
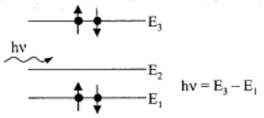
What is Optoelectronics?

Involving both electronic and optical (photonic) processes:
- Electronic excitation through photon absorption
- Photon emission through electron relaxation
- Energy converted from electrical to optical or vice versa

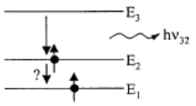


Absorption and Emission Issues

- What if E_3 occupied?



- What if E_1 has the same spin?



Classification of Optoelectronic Devices

- **Light Emitting Devices**
  - LED, Lasers, LEEC (Light-emitting electrochemical Cell)
- **Light Absorbing Devices**
  - Photodetector, Solar Cell, Photoresistor (LDR)
- **Light Manipulation Devices**
  - Involved in modulation, switching and guiding

Applications of Optoelectronic Devices

- **PhotoVoltaics**
  - Convert solar energy into electrical energy (PV)
- **Displays**
  - LCD, LED, TFT, OLED
- **Communications**
  - Optical Communication: Fibre Optics (FO), Free-Space Optics (FSO)
- **Sensing & Monitoring**
  - CCD (Charge-Coupled Device), Infrared

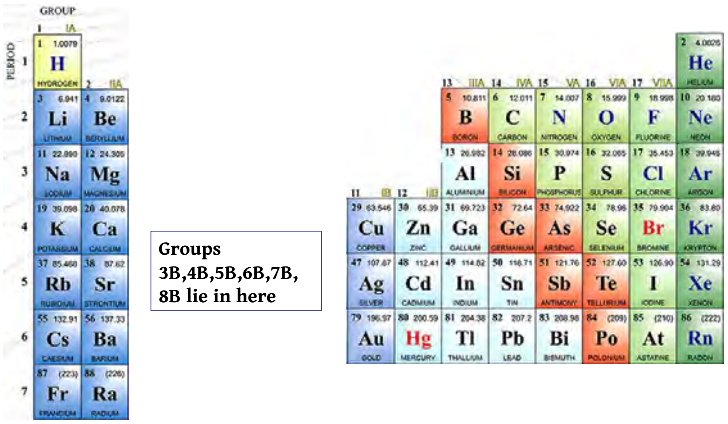
Slide Set 1

- Atoms Bonding
- Types of Solids
- Energy Bands

Atoms and Bonding

- In order to understand the physics of semiconductor (s/c) devices, we should first learn how atoms bond together to form the solids.
- Atom is composed of a nucleus which contains protons and neutrons; surrounding the nucleus are the electrons.
- Atoms can combine with themselves or other atoms. The valence electrons, i.e. the outermost shell electrons govern the chemistry of atoms.
- Atoms come together and form gases, liquids or solids depending on the strength of the attractive forces between them.
- The atomic bonding can be classified as ionic, covalent, metallic, van der Waals, etc.
- In all types of bonding the ***electrostatic force acts between charged particles.***

The Periodic Table



Groups 3B,4B,5B,6B,7B, 8B lie in here

The Periodic Table

- Ionic solids

  Group 1A (alkali metals) contains lithium (Li), sodium (Na), potassium (K),... and these combine easily with group 7A (halogens) of fluorine (F), chlorine (Cl), bromine (Br),... to produce ionic solids of NaCl, KCl, KBr, etc.

- Rare (noble) gases

  Group 8A elements of noble gases of helium(He), neon (Ne), argon (Ar),... have a full complement of valence electrons and so do not combine easily with other elements.

- Elemental semiconductors

  Silicon(Si) and Germanium (Ge) belong to group 4A.

- Compound semiconductors

1) III-V compound s/c's; GaP, InAs, AlGaAs (group 3A-5A)

2) II-VI compound s/c's; ZnS, CdS, etc. (group 2B-6A)

Covalent Bonding

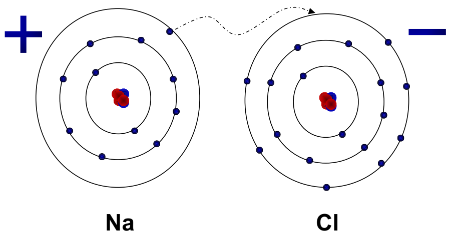
- Elemental semiconductors of Si, Ge and Carbon (as diamond) are bonded by this mechanism and these are purely covalent.
- The bonding is due to the ***sharing of electrons***.
- Covalently bonded solids are hard, high melting points, and insoluble in all ordinary solids.
- Compound s/c's exhibit a mixture of both ionic and covalent bonding.

Ionic Bonding

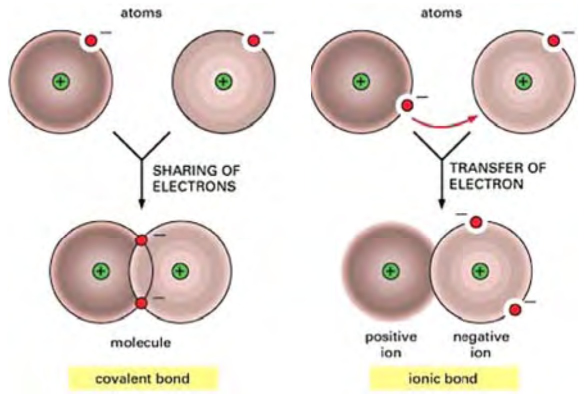
- Ionic bonding is due to the electrostatic force of attraction between positively and negatively charged ions (between 1A and 7A).
- This process leads to electron transfer and formation of charged ions; a ***positively*** charged ion for the atom that has ***lost the electron*** and a *negatively* charged ion for the atom that has ***gained an electron.***
- All ionic compounds are ***crystalline solids*** at room temperature.
- NaCl and CsCl are typical examples of ionic bonding.
- Ionic crystals are hard, high melting point, brittle and can be dissolved in ordinary liquids.

Ionic Bonding

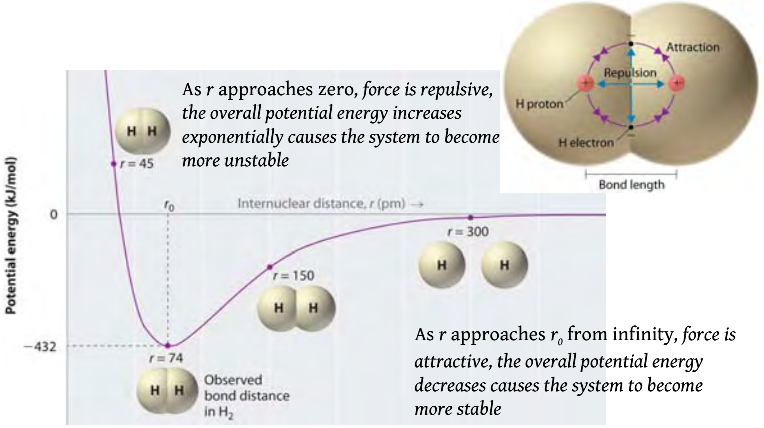
The metallic elements have only up to the valence electrons in their outer shell will lose their electrons and become positive ions, whereas electronegative elements tend to acquire additional electrons to complete their octet and become negative ions, or anions.



Covalent vs. Ionic Bonding



Potential Energy Diagram for a Hydrogen Molecule



As r approaches zero, force is repulsive, the overall potential energy increases exponentially causes the system to become more unstable

As r approaches r_0 from infinity, force is attractive, the overall potential energy decreases causes the system to become more stable

This typical curve has a minimum at equilibrium distance r_0, i.e. Bond length is 74 pm

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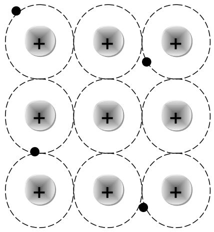
Metallic Bonding

❖ Valence electrons are relatively bound to the nucleus and therefore they move freely through the metal and they are spread out among the atoms in the form of a low-density electron cloud.

❖ A metallic bond result from the sharing of a variable number of electrons by a variable number of atoms. A metal may be described as a ***cloud of free electrons.***



❖ Therefore, metals have high electrical and thermal conductivity.

Metallic Bonding

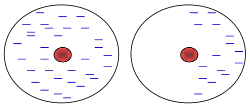
- All valence electrons in a metal combine to form a "sea" of electrons that *move freely between the atom* cores. The more electrons, the stronger the attraction. This means the melting and boiling points are higher, and the metal is stronger and harder.
- The positively charged cores are held together by these negatively charged electrons.
- The free electrons act as the bond (or as a "glue") between the positively charged ions.
- This type of bonding is non-directional and is rather insensitive to structure.
- As a result we have a high ductility of metals - the "bonds" do not "break" when atoms are rearranged – metals can experience a significant degree of plastic deformation.

Van Der Waals' Bonding

- It is the weakest bonding mechanism.
- It occurs between neutral atoms and molecules.
- The explanation of these weak forces of attraction is that there are natural fluctuation in the electron density of all molecules and these cause small temporary dipoles within the molecules. It is these temporary dipoles that attract one molecule to another. They are as called van der Waals' forces.
- Such a weak bonding results low melting and boiling points and little mechanical strength.

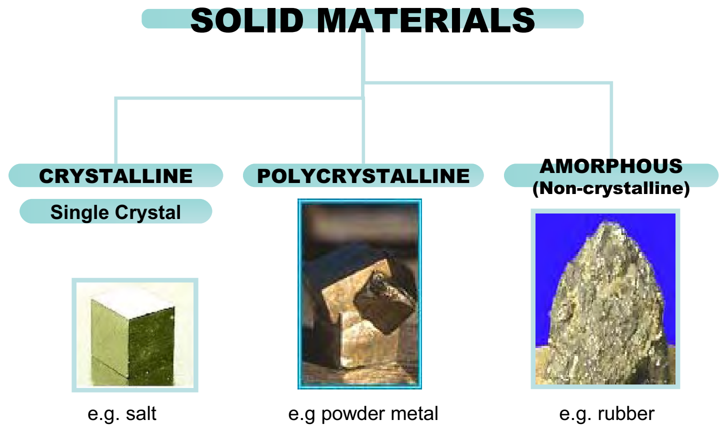
Van Der Waals' Bonding

The dipoles can be formed as a result of ***unbalanced distribution of electrons in asymettrical molecules***. This is caused by the instantaneous location of a few more electrons on one side of the nucleus than on the other.



Therefore atoms or molecules containing dipoles are attracted to each other by electrostatic forces.

Classification of Solids



SOLID MATERIALS

CRYSTALLINE

Single Crystal

e.g. salt

POLYCRYSTALLINE

e.g powder metal

AMORPHOUS
(Non-crystalline)

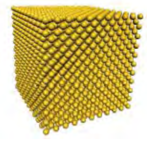
e.g. rubber

Crystalline Solid

- Crystalline Solid is the solid form of a substance in which the atoms or molecules are arranged in a definite, repeating pattern in three dimension.

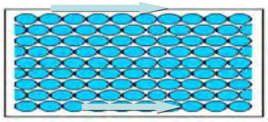
Crystalline Solid

- Single crystal has an atomic structure that repeats periodically across its whole volume. Even at infinite length scales, each atom is related to every other equivalent atom in the structure by translational symmetry



Single Crystal

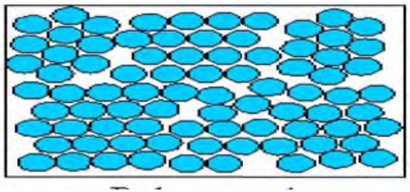
Single Pyrite Crystal



Amorphous Solid

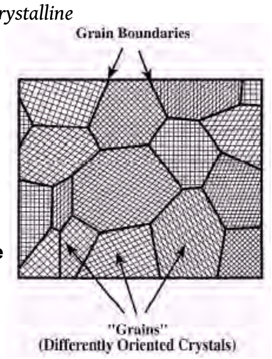
Polycrystalline Solids

- Polycrystal is a material made up of an aggregate of many small single crystals (also called crystallites or grains).
- The grains are usually 100 nm - 100 microns in diameter. Polycrystals with grains that are <10 nm in diameter are called *nanocrystalline*



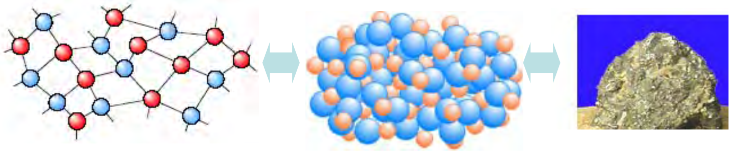
Polycrystal

Polycrystalline
→ **Pyrite form**
(Grain)

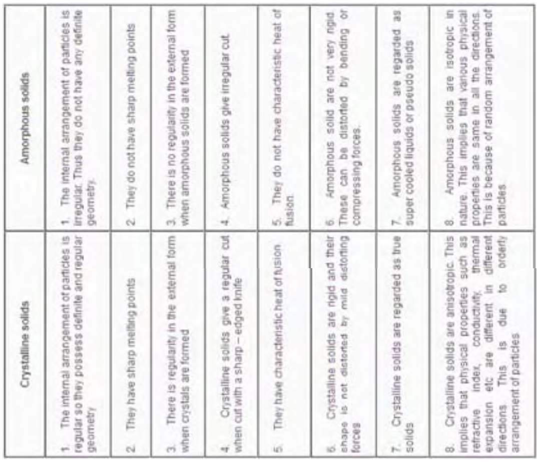


Amorphous Solids

- Amorphous (non-crystalline) Solid is composed of randomly orientated atoms, ions, or molecules that do not form defined patterns or lattice structures.



Difference between Solids



Crystalline solids	Amorphous solids
1. The internal arrangement of particles is regular so they possess definite and regular geometry	1. The internal arrangement of particles is irregular. Thus they do not have any definite geometry.
2. They have sharp melting points	2. They do not have sharp melting points
3. There is regularity in the external form when crystals are formed	3. There is no regularity in the external form when amorphous solids are formed
4. Crystalline solids give a regular cut when cut with a sharp – edged knife	4. Amorphous solids give irregular cut.
5. They have characteristic heat of fusion	5. They do not have characteristic heat of fusion.
6. Crystalline solids are rigid and their shape is not distorted by mild distorting forces	6. Amorphous solid are not very rigid. These can be distorted by bending or compressing forces.
7. Crystalline solids are regarded as true solids	7. Amorphous solids are regarded as super cooled liquids or pseudo solids
8. Crystalline solids are anisotropic. This implies that physical properties such as refractive index, conductivity, thermal expansion etc are different in different directions. This is due to orderly arrangement of particles	8. Amorphous solids are isotropic in nature. This implies that various physical properties are same in all the directions. This is because of random arrangement of particles.

Semiconductors, Insulators and Metals

The electrical properties of metals and insulators are well known to all of us.

Everyday experience has already taught us a lot about the electrical properties of metals and insulators.

But the same cannot be said about "semiconductors".

What happens when we connect a battery to a piece of a silicon;

would it conduct well ?

Or would it act like an insulator ?

Difference between Solids

The name "semiconductor" implies that it conducts somewhere between the two cases (conductors or insulators)

Conductivity : σ

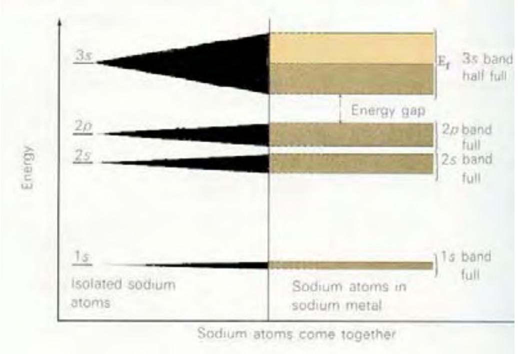
σmetals ~10^{10} /Ω-cm

S/C

σinsulators ~ 10^{-22} /Ω-cm

The conductivity (σ) of a semiconductor (S/C) lies between these two extreme cases.

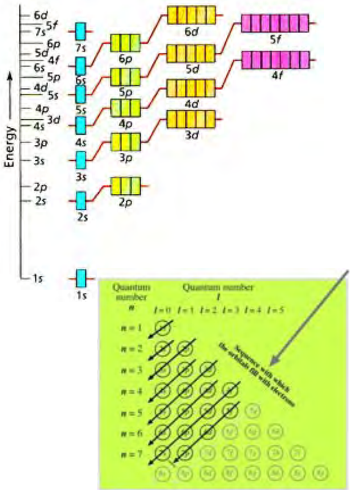
Band Theory of Solids



- The electrons orbiting around the nucleus have certain well-defined energy-levels.
- Electrons don't like to have the same energy in the same potential system.
- The most we could get together in the same energy-level was two, provided thet they had opposite spins. This is called ***Pauli Exclusion Principle.***

Difference between Solids

- The difference in energy between each of these smaller levels is so tiny that it is more reasonable to consider each of these sets of smaller energy-levels as being continuous *bands* of energy, rather that considering the enormous number of discrete individual levels.
- Each ***allowed band*** is seperated from another one by a ***forbidden band***.
- Electrons can be found in ***allowed bands*** but they can not be found in ***forbidden bands.***



Example

- ***Consider 1 cm³ of Silicon. How many atoms does this contain ?***
- Solution:

The atomic mass of silicon is 28.1 g which contains Avagadro's number of atoms.

Avagadro's number N is 6.02 x 10^{23} atoms/mol .

The density of silicon: 2.3 x 10^3 kg/m^3

so 1 cm^3 of silicon weighs 2.3 gram and so contains

$$\frac{6.02\times 10^{23}}{28.1}\times 2.3 = 4.93\times 10^{22}\,atoms$$

This means that in a piece of silicon just one cubic centimeter in volume, each electron energy-level has split up into 4.93 x 10^{22} smaller levels !

Semiconductor, Insulators, Conductors

Full band

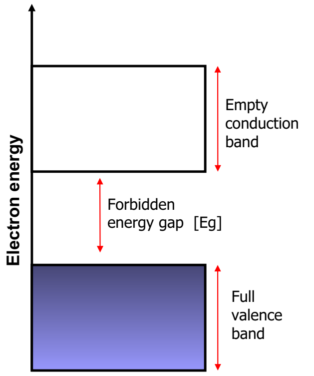
All energy levels are occupied by electrons

Empty band

All energy levels are empty (no electrons)

Both full and empty bands do not partake in electrical conduction.

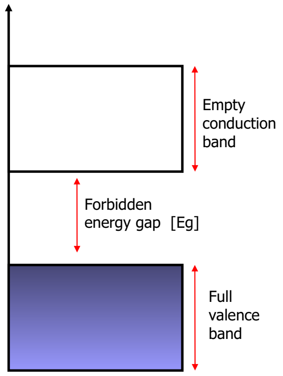
Semiconductor energy bands at low temperature



- At low temperatures the valence and the conduction band is empty.
- Recall that a full band can not conduct, and neither can an empty band.
- At low temperatures, s/c's do not conduct, they behave like insulators.
- The ***thermal energy*** of the electrons sitting at the top of the full band is much lower than that of the ***Eg*** *at low temperatures.*

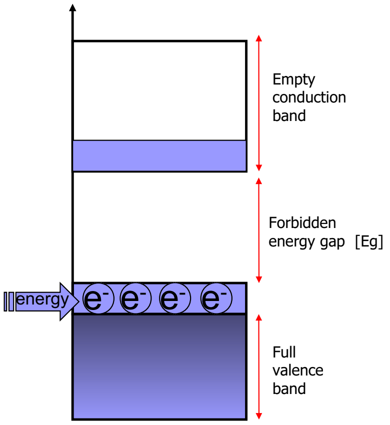
Conduction Electrons

- Assume some kind of energy is provided to the electron (valence electron) sitting at the top of the *valence band.*
- This electron gains energy from the applied field and it would like to move into higher energy states.
- This electron contributes to the ***conductivity*** and this electron is called as a ***conduction electron***.
- At 0^0K, electron sits at the lowest energy levels. The valence band is the highest filled band at zero kelvin.



Semiconductor energy bands at room temperature

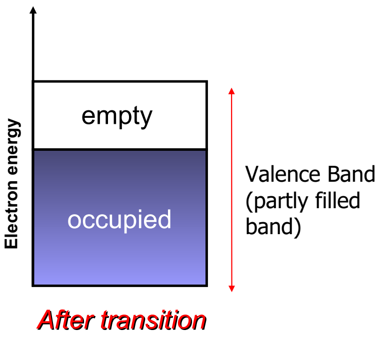
- When enough ***energy*** is supplied to the ***e^-*** sitting at the top of the valence band, ***e^-*** can make a transition to the bottom of the conduction band.
- When electron makes such a transition it leaves behind a ***missing electron state.***
- This missing electron state is called as a ***hole.***
- Hole behaves as a ***positive charge carrier.***
- Magnitude of its charge is the same with that of the electron but with an opposite sign.



Conclusion

- Holes contribute to current in ***valence band*** (VB) as electrons are able to create current in ***conduction band*** (CB).
- Hole is ***not*** a free particle. It can only exist within the crystal. A hole is simply a vacant electron state.
- A transition results an equal number of e^- in CB and holes in VB. This is an important property of *intrinsic* or *undoped* semiconductors. For *extrinsic* or *doped* semiconductors this is no longer true.

Bipolar (two carrier) conduction

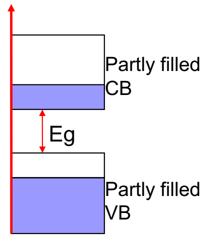


- ***After transition***, the valence band is now no longer full, it is *partly filled* and may conduct electric current.
- The ***conductivity*** is due to both electrons and holes, and this device is called a ***bipolar conductor*** or ***bipolar device.***

What kind of excitation mechanism can cause an e^- to make a transition from the top of the valance band (VB) to the minimum or bottom of the conduction band (CB) ?

Answer :

- Thermal energy ?
- Electrical field ?
- Electromagnetic radiation ?



Energy band diagram of a s/c at a finite temperature.

To have a partly field band configuration in a s/c, one must use one of these excitation mechanisms.

1 - Thermal Energy

Thermal energy at RTP = k x T = 1.38 x 10^{-23} J/K x 298 K = 25.7 meV

Excitation rate = *constant* x *exp(-Eg / kT)*

Although the thermal energy at room temperature is very small, *i.e.* 25 meV, a few electrons can be promoted to the CB.

Electrons can be promoted to the CB by means of thermal energy.

This is due to the exponential increase of excitation rate with increasing temperature.

Excitation rate is a strong function of temperature.

2 - Electric Field

- For low fields, this mechanism doesn't promote electrons to the CB in common s/c's such as Si and GaAs.
- An electric field of 10^{18} V/m can provide an energy of the order of 1 eV. This field is enormous.

So , the use of the electric field as an excitation mechanism is not useful way to promote electrons in s/c's.

3- Electromagnetic Radiation

$$E = h\nu = h\frac{c}{\lambda} = (6.62x10^{-34} J - s)x(3x10^{8} m/s)/\lambda\,(m) \Rightarrow E(eV) = \frac{1.24}{\lambda\,(\text{in}\ \mu\, m)}$$

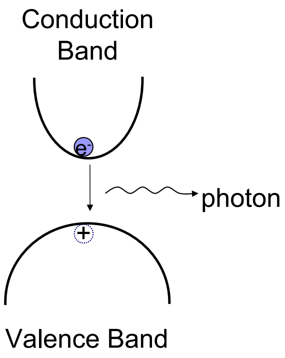
$h = 6.62 \text{ x } 10^{-34}$ Js
$c = 3x10^{8}$ m/s
1 eV=$1.6x10^{-19}$ J

Near infrared

$$for\quad \text{Silicon}\quad E_g = 1.1eV \qquad \lambda\,(\mu\, m) = \frac{1.24}{1.1} = 1.1\mu\, m$$

To promote electrons from VB to CB Silicon , the wavelength of the photons must 1.1 μm or less

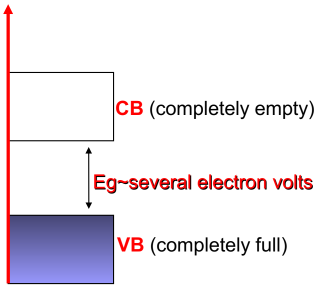
Direct Band-Gap



- The converse transition can also happen.
- An electron in CB recombines with a hole in VB and generate a photon.
- The energy of the photon will be in the order of Eg.
- If this happens in a direct band-gap s/c, it forms the basis of LED's and LASERS.

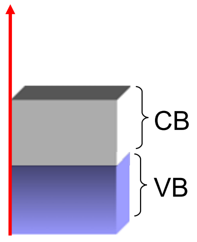
Insulators

- The magnitude of the band gap determines the differences between insulators, s/c's and metals.
- The excitation mechanism of thermal is not a useful way to promote an electron to CB even the melting temperature is reached in an insulator.
- Even very high electric fields is also unable to promote electrons across the band gap in an insulator.

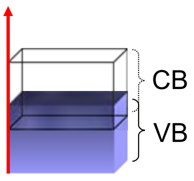


Wide band gaps between VB and CB

Metals



Touching VB and CB

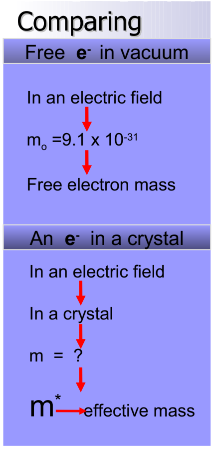


Overlapping VB and CB

- These two bands looks like as if partly filled bands and it is known that partly filled bands conducts well.
- This is the reason why metals have high conductivity.

- No gap between ***valance band*** and ***conduction band***

The Concept of Effective Mass


Comparing

Free e⁻ in vacuum

In an electric field

$m_o = 9.1 \times 10^{-31}$

Free electron mass

An e⁻ in a crystal

In an electric field

In a crystal

m = ?

m^* effective mass

- If the same magnitude of electric field is applied to both electrons in vacuum and inside the crystal, the electrons will accelerate at a different rate from each other due to the existence of different potentials inside the crystal.
- The electron inside the crystal has to try to make its own way.
- So the electrons inside the crystal will have a different mass than that of the electron in vacuum.
- This altered mass is called as an ***effective-mass***.

What is the expression for m*

- Particles of electrons and holes behave as a wave under certain conditions. So one has to consider the *de Broglie* wavelength to link partical behaviour with wave behaviour.
- Particles such as electrons and waves can be diffracted from the crystal just as X-rays .
- Certain electron momentum is not allowed by the crystal lattice. This is the origin of the energy band gaps.

$$n\lambda = 2d\sin\theta$$

n = the order of the diffraction
λ = the wavelength of the X-ray
d = the distance between planes
θ = the incident angle of the X-ray beam

$$n\lambda = 2d \quad (1)$$

The waves are standing waves

$$\lambda = \frac{2\pi}{k}$$

k is the propagation constant

The momentum is

$$P = \hbar k \quad (2)$$

The energy of the free electron can be related to its momentum

$$E = \frac{P^2}{2m} \qquad P = \frac{h}{\lambda}$$

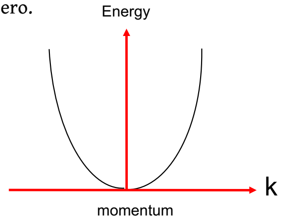
free e⁻ mass , m_0

$$E = \frac{h^2}{2m}\frac{1}{\lambda^2} = \frac{h^2}{2m}\frac{k^2}{(2\pi)^2}$$

$$\hbar = \frac{h}{2\pi} \qquad E = \frac{\hbar^2 k^2}{2m}$$

The energy of the free e⁻ is related to the k

By means of equations (1) and (2) certain e⁻ momenta are not allowed by the crystal. The velocity of the electron at these momentum values is zero.



E versus k diagram is a parabola.

Energy is continuous with k, i,e, all energy (momentum) values are allowed.

E versus k diagram
or
Energy versus momentum diagrams

To find effective mass , m*

We will take the derivative of ***energy*** with respect to ***k ;***

$$\frac{dE}{dk} = \frac{\hbar^2 k}{m}$$

$$\frac{d^2E}{dk^2} = \frac{\hbar^2}{m}$$

- ***m**** is determined by the curvature of the E-k curve
- ***m**** is inversely proportional to the curvature

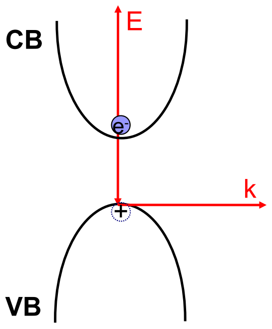
Change ***m**** instead of ***m***

$$m^* = \frac{\hbar^2}{d^2E/dk^2}$$

This formula is the ***effective mass*** of an electron inside the crystal.

Direct-band gap materials

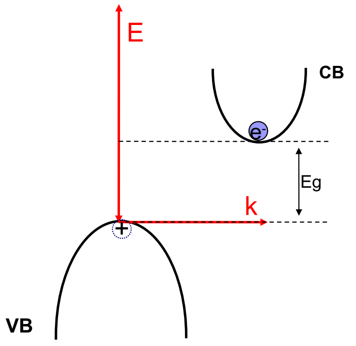
Direct-band gap s/c's (e.g. GaAs, InP, AlGaAs)



- For a ***direct-band gap material***, the minimum of the conduction band and maximum of the valence band lies at the same momentum, k, values.
- When an electron sitting at the bottom of the CB recombines with a hole sitting at the top of the VB, there will be no change in momentum values.
- Energy is conserved by means of emitting a photon, such transitions are called as radiative transitions.

Indirect-band gap materials

Indirect-band gap s/c's (e.g. Si and Ge)



- For an indirect-band gap material; the minimum of the CB and maximum of the VB lie at different k-values.
- When an e⁻ and hole recombine in an indirect-band gap s/c, phonons must be involved to conserve momentum.

Phonon

- Atoms vibrate about their mean position at a finite temperature.These vibrations produce vibrational waves inside the crystal.
- Phonons are the quanta of these vibrational waves. Phonons travel with a velocity of sound .
- Their wavelength is determined by the crystal lattice constant. Phonons can only exist inside the crystal.

Conclusion

- The transition that involves phonons without producing photons are called *non-radiative (radiationless) transitions*.
- These transitions are observed in an *indirect band gap* s/c and result in inefficient photon producing.
- So in order to have efficient LED's and LASER's, one should choose materials having direct band gaps such as compound s/c's of GaAs, AlGaAs, etc...

Calculation

- *For GaAs, calculate a typical (band gap) photon energy and momentum , and compare this with a typical phonon energy and momentum that might be expected with this material.*

photon	phonon
E(photon) = Eg(GaAs) = 1.43 ev	E(phonon) = h υ= hv$_s$ / λ = hv$_s$ / a_0
E(photon) = h υ= hc / λ	λ (phonon) ~a$_0$ = lattice constant =5.65x10^{-10} m
c= 3x10^8 m/sec	V_s= 5x10^3 m/sec (velocity of sound)
P = h / λ h=6.63x10^{-34} J-sec	E(phonon) = hv$_s$ / a$_0$ =0.037 eV
λ (photon)= 1.24 / 1.43 = 0.88 μm	P(phonon)= h / λ = h / a$_0$ = 1.17x10^{-24} kg-m/sec
P(photon) = h / λ = 7.53 x 10^{-28} kg-m/sec	

Phonons vs Photons

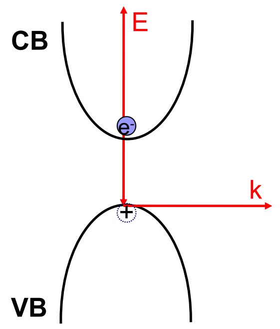
- Photon energy = 1.43 eV
- Phonon energy = 37 meV
- Photon momentum = 7.53 x 10^{-28} kg-m/sec
- Phonon momentum = 1.17 x 10^{-24} kg-m/sec

Photons carry large energies but negligible amount of momentum.

On the other hand, phonons carry very little energy but significant amount of momentum.

Positive and Negative effective mass

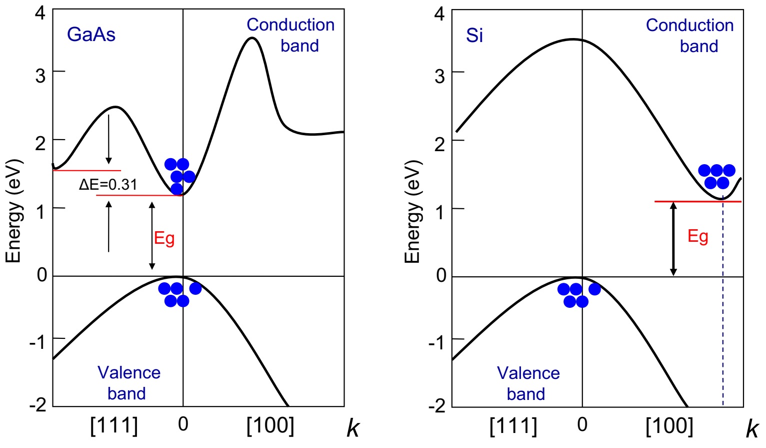
Direct-band gap s/c's (e.g. GaAs, InP, AlGaAs)



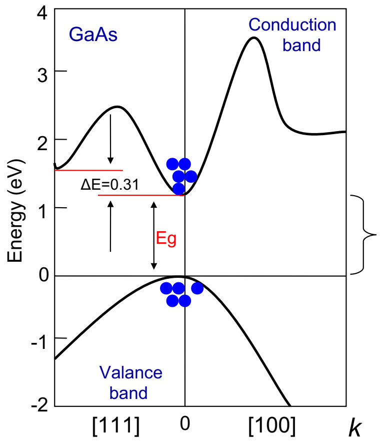
$$m^* = \frac{h^2}{d^2E/dk^2}$$

- The sign of the effective mass is determined directly from the sign of the curvature of the E-k curve.
- The curvature of a graph at a minimum point is a positive quantity and the curvature of a graph at a maximum point is a negative quantity.
- Particles(electrons) sitting near the minimum have a ***positive effective mass***.
- Particles(holes) sitting near the valence band maximum have a ***negative effective mass***.
- A negative effective mass implies that a particle will go ***'the wrong way'*** when an extrernal force is applied.

Energy band structures of *GaAs* and *Si*



Energy band structure of *GaAs*



Band gap is the smallest energy separation between the valence and conduction band edges.

The smallest energy difference occurs at the same momentum value

⇩

Direct-band gap semiconductor

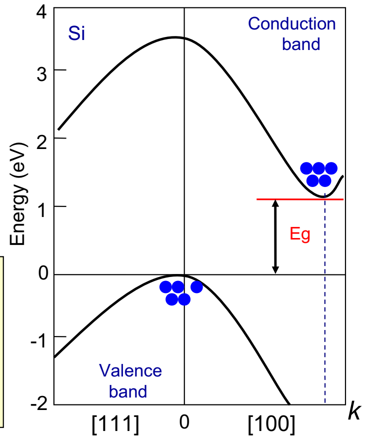
Energy band structure of *Si*

The smallest energy gap is between the top of the VB at k=0 and one of the CB minima away from k=0

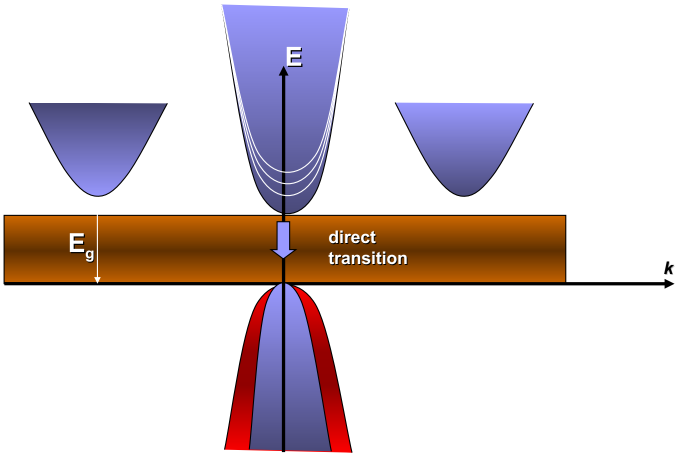
⇩

Indirect band gap semiconductor

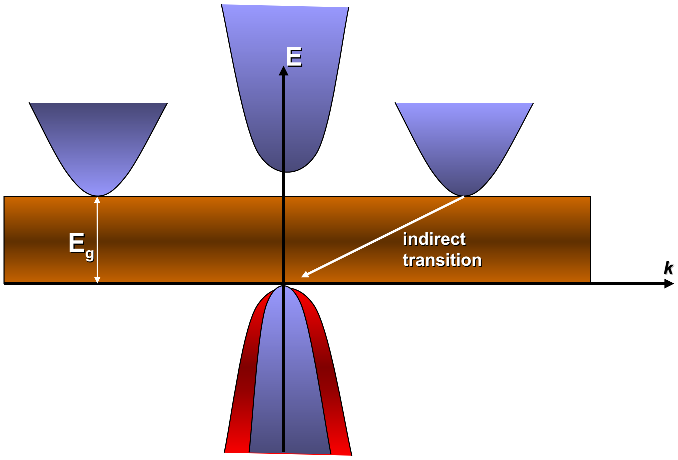
- •Band structure of AlGaAs?
- •Effective masses of CB satellites?
- •Heavy- and light-hole masses in VB?



Direct transition



Indirect transition



Slide Set 2

Carrier Concentration in Semiconductors

CARRIER CONCENTRATIONS IN SEMICONDUCTORS

Donors and Acceptors

Fermi level , E_f

Carrier concentration equations

Donors and acceptors both present

Donors and Acceptors

The conductivity of a pure (intrinsic) s/c is low due to the low number of free carriers.

- The number of carriers are generated by thermally or electromagnetic radiation for a pure s/c.

- For an intrinsic semiconductor

$$n = p = n_i$$

n = concentration of electrons per unit volume
p = concentration of holes per unit volume
n_i = the intrinsic carrier concentration of the semiconductor under consideration.

$n.p = n_i^2$

n = p

number of e⁻'s in CB = number of holes in VB

- This is due to the fact that when an e⁻ makes a transition to the CB, it leaves a hole behind in VB. We have a bipolar (two carrier) conduction and the number of holes and e⁻'s are equal.

$$n.p = n_i^2$$

This equation is known as **mass-action law.**

$n.p = n_i^2$

The intrinsic carrier concentration n_i depends on; the semiconductor material, and the temperature.

- For silicon at 300 K, n_i has a value of 1.4×10^{10} cm^{-3}.
- Clearly , equation ($n = p = n_i$) can be written as

$$n.p = n_i^2$$

- This equation is valid for **extrinsic** as well as **intrinsic** material.

What are doping and dopants impurities ?

To increase the conductivity, one can dope pure s/c with atoms from column lll or V of periodic table. This process is called as doping and the added atoms are called as dopants impurities.

There are two types of doped or extrinsic s/c's;

- n-type
- p-type

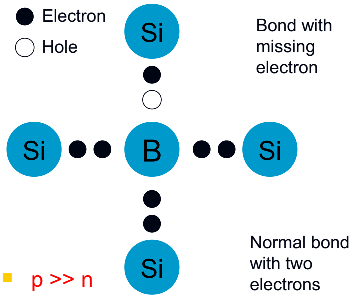
Addition of different atoms modify the conductivity of the intrinsic semiconductor.

P-type doped semiconductor

- Si + Column III impurity atoms

Have four valence electrons

Boron (B) has three valence electrons



- Boron bonding in Silicon
- Boron sits on a lattice side

- p >> n

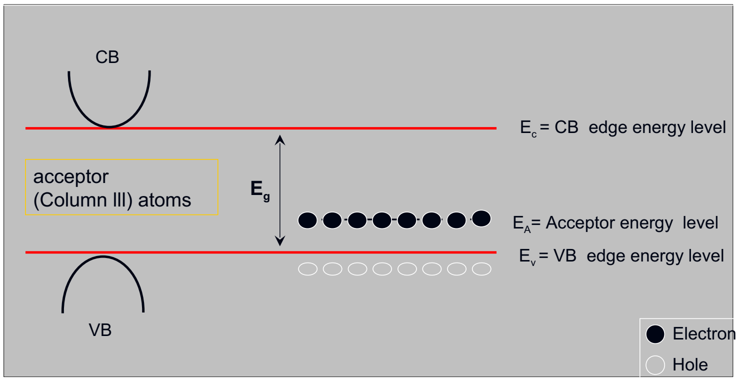
P-type doped semiconductor

Boron (column III) atoms have three valence electrons, there is a deficiency of electron or missing electron to complete the outer shell.

This means that each added or doped Boron atom introduces a single hole in the crystal.

- There are two ways of producing holes
  1) Promote electrons from VB to CB,
  2) Add column lll impurities to the s/c.

Energy Diagram for a P-type s/c



The energy gap is forbidden only for pure material, i.e. Intrinsic material.

P-type semiconductor

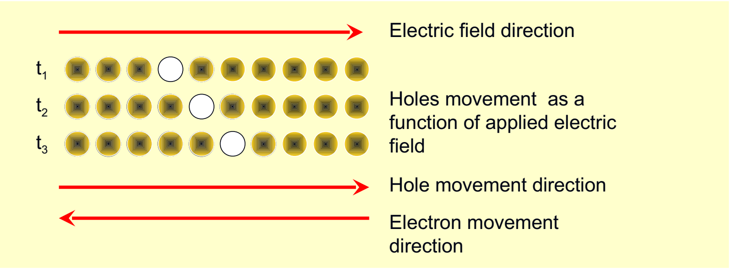
- ***The impurity atoms from column lll occupy at an energy level within E_g These levels can be:***

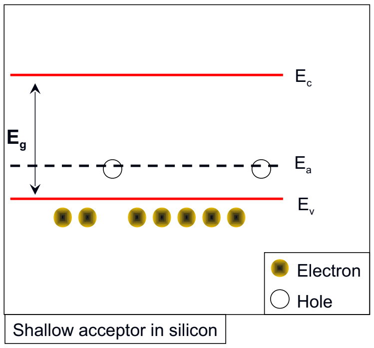
1. Shallow levels which is close to the band edge,
2. Deep levels which lies almost at the mid of the band gap.

- If the E_A level is shallow i.e. close to the VB edge, each added boron atom accepts an e^- from VB and have a full configuration of e^-'s at the outer shell.
- These atoms are called as acceptor atoms since they accept an e^- from VB to complete its bonding. So each acceptor atom gives rise a hole in VB.
- The current is mostly due to holes since the number of holes are made greater than e^-'s.

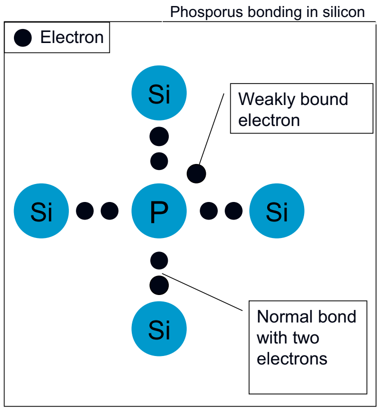
Majority & Minority carrier in a P-type semiconductor

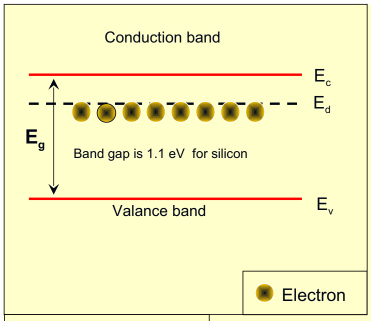
Holes = p = majority carriers
Electrons = n = minority carriers



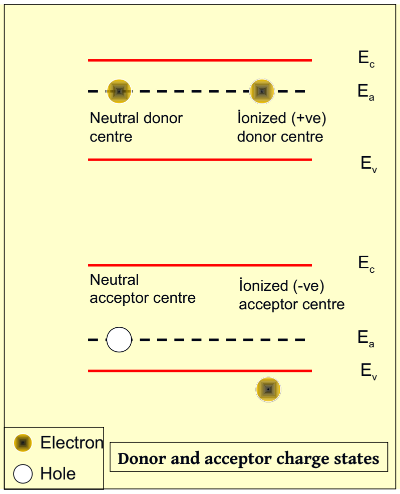


Shallow acceptor in silicon



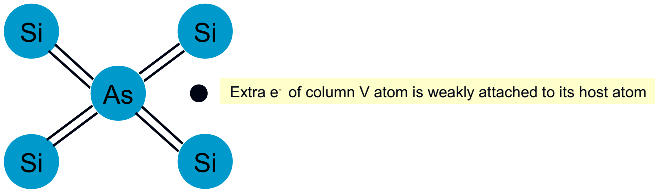


Shallow donor in silicon

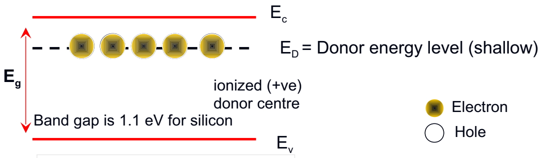


Donor and acceptor charge states

N-type doped semiconductor



Si + column V (with five valance e^-)



n - type semiconductor

Np vs. Pn semiconductors

n-type , $n >> p$; n is the majority carrier concentration n_n
p is the minority carrier concentration p_n

p-type , $p >> n$; p is the majority carrier concentration p_p
n is the minority carrier concentration n_p

n_p p_n

Type of semiconductor

Calculation

Calculate the hole and electron densities in a piece of P-type silicon that has been doped with 5 x 10^{16} acceptor atoms per cm^3.

n_i = 1.4 x 10^{10} cm^{-3} (at room temperature)

Undoped

$n = p = n_i$

P-type ; p >> n

$n.p = n_i^2$ N_A = 5 x 10^{16} $p = N_A$ = 5 x 10^{16} cm^{-3}

$$n = \frac{n_i^2}{p} = \frac{(1.4x10^{10}cm^{-3})^2}{5x10^{16}cm^{-3}} = 3.9x10^3$$ electrons per cm^3

p >> n_i and n << n_i in a p-type material. The more holes you put in the less electrons you have and vice versa.

Fermi level , E_F

This is a reference energy level at which the probability of occupation by an electron is ½.

Since E_f is a reference level therefore it can appear anywhere in the energy level diagram of a S/C .

Fermi energy level is not fixed.

Occupation probability of an electron and hole can be determined by Fermi-Dirac distribution function, F_{FD} ;

$$F_{FD} = \frac{1}{1 + \exp(\frac{E - E_F}{k_B T})}$$

E_F = Fermi energy level

k_B = Boltzman constant

T = Temperature

Fermi level , E_F

$$F_{FD} = \frac{1}{1 + \exp(\frac{E - E_F}{k_B T})}$$

E is the energy level under investigation.

F_{FD} determines the probability of the energy level E being occupied by electron.

$$if\ E = E_F \longrightarrow f_{FD} = \frac{1}{1 + \exp 0} = \frac{1}{2}$$

$1 - f_{FD}$ determines the probability of not finding an electron at an energy level E; the probability of finding a hole .

Carrier concentration equations

The number density, i.e., the number of electrons available for conduction in CB is

$$n = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2} \exp - (\frac{E_C - E_F}{kT})$$

$$n = N_C \exp - (\frac{E_C - E_F}{kT}) \qquad n = n_i \exp(\frac{E_F - E_i}{kT})$$

The number density, i.e., the number of holes available for conduction in VB is

$$p = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2} \exp - (\frac{E_F - E_V}{kT})$$

$$p = N_V \exp - (\frac{E_F - E_V}{kT}) \qquad p = n_i \exp(\frac{E_i - E_F}{kT})$$

Donors and acceptors both present

Both donors and acceptors present in a s/c in general. However one will outnumber the other one.

In an n-type material the number of donor concentration is significantly greater than that of the acceptor concentration.

Similarly, in a p-type material the number of acceptor concentration is significantly greater than that of the donor concentration.

A p-type material can be converted to an n-type material or vice versa by means of adding proper type of dopant atoms. This is in fact how p-n junction diodes are actually fabricated.

Example

- How does the position of the Fermi Level change with

(a) increasing donor concentration, and

(b) increasing acceptor concentration ?

(a) We shall use equation

$$n = N_C \exp - (\frac{E_C - E_F}{kT})$$

If n is increasing then the quantity E_C-E_F must be decreasing i.e. as the donor concentration goes up the Fermi level moves towards the conduction band edge E_C.

Example

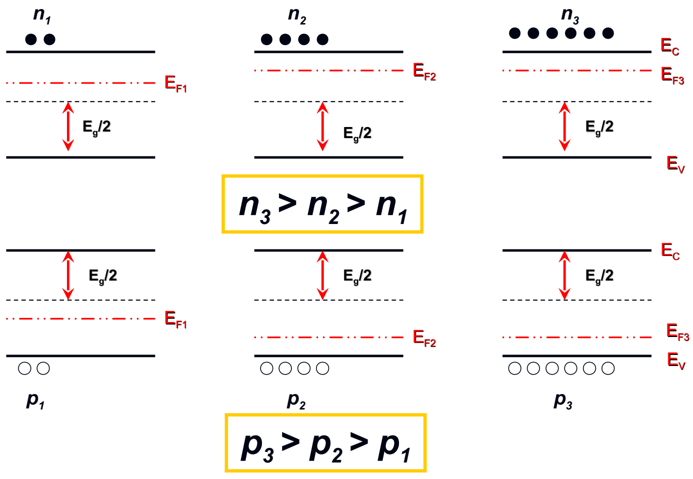
But the carrier density equations such as;

$$n = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{\frac{3}{2}} \exp-\left(\frac{E_c - E_F}{kT}\right) \quad and$$

$$p = n_i \exp\left(\frac{E_i - E_F}{kT}\right)$$

aren't valid for all doping concentrations! As the fermi-level comes to within about **3kT** of either band edge the equations are no longer valid, because they were derived by assuming the simpler **Maxwell Boltzmann statistics** rather than the proper Fermi-Dirac statistic.

Example



Example

(b) Considering the density of holes in valence band;

$$p = N_v \exp-\left(\frac{E_F - E_V}{kT}\right)$$

It is seen that as the acceptor concentration increases, Fermi-level moves towards the *valance band edge.* These results will be used in the construction of device (energy) band diagrams.

Donors and acceptor both present

- In general, both donors and acceptors are present in a piece of a semiconductor although one will outnumber the other one.
- The impurities are incorporated unintentionally during the growth of the semiconductor crystal causing both types of impurities being present in a piece of a semiconductor.
- How do we handle such a piece of s/c?

1) Assume that the shallow donor concentration is significantly greater than that of the shallow acceptor concentration. In this case the material behaves as an n-type material and

$$n_n = N_D - N_A$$

2) Similarly, when the number of shallow acceptor concentration is signicantly greater than the shallow donor concentration in a piece of a s/c, it can be considered as a p-type s/c and

$$p_p = N_A - N_D$$

Donors and acceptor both present

For the case **$N_A > N_D$, i.e.** for p-type material

$$n_p . p_p = n_i^2$$

$$n_p + N_A^- = N_D^+ + p_P \Rightarrow p_p + N_D - n_p - N_A = 0$$

$$p_p \times \left[p_p + N_D - \frac{n_i^2}{p_p} - N_A = 0 \right] \Rightarrow \boxed{p_p^2 + (N_D - N_A)p_p - n_i^2 = 0}$$

Donors and acceptor both present

$$\boxed{p_p^2 + (N_D - N_A)p_p - n_i^2 = 0}, \text{ solving for } p_p \,;\, x_{1,2} = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a}$$

$$p_p = \frac{1}{2}\left(N_A - N_D + \left[\left(N_A - N_D \right)^2 + 4n_i^2 \right]^{1/2} \right) \;;$$ majority

$$n_p = \frac{n_i^2}{p_p}$$ minority

Donors and acceptor both present

For the case **$N_D > N_A$** , i.e. n-type material

$$n_n . p_n = n_i^2 \Rightarrow p_n = \frac{n_i^2}{n_n}$$

$$n_n + N_A = N_D + P_n \;\Rightarrow n_n + N_A - p_n - N_D = 0$$

$$n_n \times \left[n_n + N_A - \frac{n_i^2}{n_n} - N_D = 0 \right] \Rightarrow n_n^2 + (N_A - N_D) n_n - n_i^2 = 0$$

$$\text{solving for } \mathrm{n}_n \text{ ; } x_{1,2} = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a}$$

$$n_n = \frac{1}{2}\left(N_D - N_A + \left[(N_D - N_A)^2 + 4n_i^2 \right]^{1/2} \right)$$

$$p_n = \frac{n_i^2}{n_n}$$

Slide Set 3

- Carrier drift
- Carrier mobility
- Saturated drift velocity
- Mobility variation with temperature
- Drift current equations
- Band diagrams with an electric field present
- Carrier diffusion
- Flux equation
- Einstein relation
- Total current density
- Carrier recombination and diffusion length

Drift and Diffusion

We now have some idea of the number density of charge carriers (electrons and holes) present in a semiconductor material from the last slide set. Since current is the rate of flow of charge, we shall be able calculate currents flowing in real devices since we know the number of charge carriers. There are two current mechanisms which cause charges to move in semiconductors. These two mechanisms are *drift and diffusion*.

Carrier Drift

- Electron and holes will move under the influence of an applied electric field since the field exert a force on charge carriers (electrons and holes).

$$F = qE$$

- These movements result a current of I_d

$$I_d = nqV_d A$$

I_d : drift current

V_d : drift velocity of charge carrier

n : number of charge carriers per unit volume

q : charge of the electron

A : area **presented** in the direction of the flow of charge carriers

---

Carrier Mobility μ

$$V_d = \mu E$$

E : applied field

μ : mobility of charge carrier

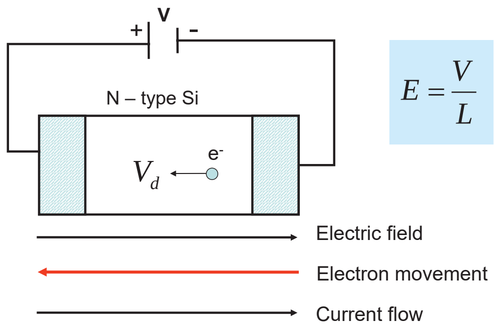
$$[\mu] = \left[\frac{cm^2}{V\,s}\right]$$

μ is a proportionality factor

$$\mu = \left[\frac{V_d}{E}\right]$$

❖ So μ is *a measure how easily charge carriers move* under the influence of an applied field or μ determines *how mobile the charge carriers* are.

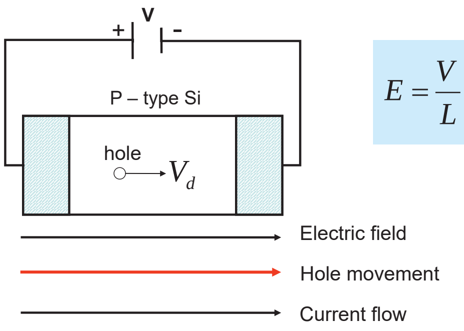
N - type Silicon



$$E = \frac{V}{L}$$

Current carriers are mostly electrons.

P - type Silicon



$$E = \frac{V}{L}$$

Current carriers are mostly holes.

Carrier Mobility

Macroscopic understanding

$$\boxed{\mu = \frac{V_d}{E}}$$

In a perfect Crystal

$$\rho = 0$$

$$\sigma \rightarrow \infty$$

It is a superconductor

Microscopic understanding? (what the carriers themselves are doing?)

$$\mu = \frac{q\tau}{m^*}$$

$m_e^* < m_h^*$ in general

m_e^* : N-type
m_h^* : P-type

μ

- A *perfect crystal has a perfect periodicity* and therefore the potential seen by a carrier in a perfect crystal is completely periodic.
- So the *crystal has no resistance to current flow* and behaves as a *superconductor*. The perfect periodic potential does not *impede the movement of the charge carriers*. However, in a real device or specimen, the presence of impurities, interstitials, subtitionals, temperature , etc. creates a resistance to current flow.
- The presence of all these *upsets the periodicity of the potential* seen by a charge carrier.

Mobility has two components

Mobility has two components

Lattice interaction component

Impurity interaction component

Thermal Velocity

- Assume that s/c crystal is at thermodynamic equilibrium (i.e. there is no applied field). What will be the energy of the electron at a finite temperature?
- The electron will have a thermal energy of kT/2 per degree of freedom. So , in 3D, electron will have a thermal energy of

$$E = \frac{3kT}{2} \Rightarrow \frac{1}{2}m^* v_{th}^2 = \frac{3kT}{2} \Rightarrow \boxed{V_{th} = \sqrt{\frac{3kT}{m^*}}}$$

V_{th} : thermal velocity of electron or hole
V_{th} is proportional to $T^{1/2}$
V_{th} is inversely proportional to $(m^*)^{1/2}$

Random motion result no current.

- Since there is no applied field, the movement of the charge carriers will be completely random. This randomness result **no net current flow**. As a result of thermal energy there are almost an equal number of carriers moving right as left, in as out or up as down.

Calculation

Calculate the velocity of an electron in a piece of n-type silicon due to its thermal energy at RT and due to the application of an electric field of 1000 V/m across the piece of silicon.

$V_{th} = ?$ $RT = 300\ K$ $m_e^* = 1.18\ m_0$
$V_d = ?$ $E = 1000\ V/m$ $\mu = 0.15\ m^2/(V-s)$

$$V_{th} = \sqrt{\frac{3kT}{m^*}} \Rightarrow \boxed{V_{th} = 1.08x10^5\ m/\sec}$$

$$V_d = \mu E \Rightarrow \boxed{V_d = 150\ m/\sec}$$

Microscopic understanding of mobility?

How long does a carrier move in time before collision ?

The average time taken between collisions is referred to as relaxation time or mean free time: τ (small Greek symbol Tau)

How far does a carrier move in space before a collision?

The average distance taken between collisions is called as mean free path: ζ (small Greek symbol Zeta)

Calculation

Drift velocity=Acceleration x Mean free time

$$V_d = \frac{F}{m^*} \times \tau$$

Force is due to the applied field, F=qE

$$V_d = \frac{F}{m^*} \times \tau = \frac{qE}{m^*} \tau$$

$$V_d = \mu E \Rightarrow \mu = \frac{q\tau}{m^*}$$

Calculation

Calculate the mean free time and mean free path for electrons in a piece of N-type silicon and for holes in a piece of P-type silicon.

$$\tau = ? \qquad l = ? \quad m_e^* = 1.18\, m_o \qquad m_h^* = 0.59 m_o$$

$$\mu_e = 0.15\, m^2/(V-s) \qquad \mu_h = 0.0458\, m^2/(V-s)$$

$$\tau_e = \frac{\mu_e m_e^*}{q} = 10^{-12} \sec \qquad \tau_h = \frac{\mu_h m_h^*}{q} = 1.54x10^{-13} \sec$$

$$v_{th_{elec}} = 1.08x10^5 m/s \qquad v_{th_{hole}} = 1.\ 52x10^5 m/s$$

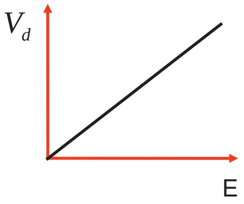
$$l_e = v_{th_{elec}} \tau_e = (1.08x10^5 m/s)(10^{-12} s) = 10^{-7} m$$

$$l_h = v_{th_{hole}} \tau_h = (1.\ 52x10^5 m/s)(1.54x10^{-13} \sec) = 2.34x10^{-8} m$$

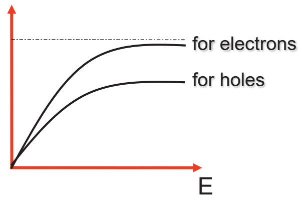
Saturated Drift Velocities

$$V_d = \mu E$$

So one can make a carrier go as fast as we like just by increasing the electric field beyond the speed of light ☺



(a) implication of above eqn.

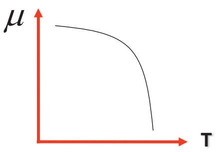


(b) Saturation drift velocity

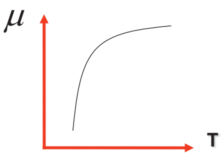
Saturated Drift Velocities

- The equation of $V_d = \mu E$ does not imply that **V_d** increases linearly with applied field **E.**
- **V_d** increases linearly for low values of **E** and then it saturates at some value of **V_d** which is close **V_{th}** at higher values of **E**.
- Any further increase in **E** after saturation point does not increase **V_d** instead warms up the crystal.

Mobility variation with Temperature



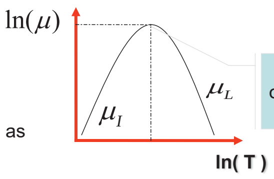
High temperature



Low temperature

$$\frac{1}{\mu_T} = \frac{1}{\mu_L} + \frac{1}{\mu_I}$$

This equation is called as Mattheisen's rule.



Peak depends on the density of impurities

Variation of mobility with temperature

At high temperature as the lattice warms up → μ_L component becomes significant.

μ_L *decreases* when temperature *increases.*

$$\mu_L = C_1 \times T^{-\frac{3}{2}} \Rightarrow T^{-\frac{3}{2}}$$

C_1 is a constant.

It is called as a $T^{-1.5}$ *power law.*

Carriers are more likely scattered by the lattice atoms.

Variation of mobility with temperature

At low temperatures → μ_I component is significant.

μ_I *decreases* when temperature *decreases.*

$$\mu_I = C_2 \times T^{\frac{3}{2}}$$

C_2 is a constant.

Carriers are more likely scattered by ionized impurities.

Variation of mobility with temperature

The peak of the mobility curve depends on the number density of ionized impurities.

Highly doped samples will therefore cause more scattering, and have a lower mobility, than low doped samples.

This fact is used in high speed devices called High Electron Mobility Transistors (HEMTs) where electrons are made to move in undoped material, with the resulting high carrier mobilities!

HEMTs are high speed devices.

A Derivation of Ohm's Law

$I_d = nqV_dA$

$V_d = \mu E$

$J_d = \frac{I_d}{A}$

$\mu = \frac{q\tau}{m^*}$

$J_x = nqV_d = nq\mu E$

$J_x = \left(\frac{nq^2\tau}{m^*}\right)E_x$

$\sigma = \frac{nq^2\tau}{m^*}$

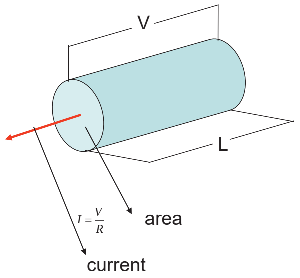
$J_x = \sigma E_x$

$\rho = \frac{1}{\sigma}$

$[\rho] = [\Omega - m]$

$[\sigma] = [1/(\Omega - m)]$

A Derivation of Ohm's Law



This is in fact ohm's law which is written slightly in a different form.

$$J_x = \sigma E_x \qquad \frac{I_x}{A} = \sigma\frac{V}{L} \Rightarrow \frac{I_x}{A} = \frac{1}{\rho}\frac{V}{L} \qquad I = \frac{VA}{\rho L} = \frac{V}{R}$$

Drift Current Equations

For undoped or intrinsic semiconductor ; $n=p=n_i$

For electron

$$J_n = nqE\mu_n$$

drift current for electrons

number of free electrons per unit volume

mobility of electron

For hole

$$J_p = pqE\mu_p$$

drift current for holes

number of free holes per unit volume

mobility of holes

Drift Current Equations

Total current density

$$J_i = J_e + J_h$$

$$J_i = nqE\mu_n + pqE\mu_P$$

since $n = p = n_i$

$$J_i = n_i q(\mu_n + \mu_p)E$$

For a pure intrinsic semiconductor

Drift Current Equations

$J_{total} = ?$ for doped or extrinsic semiconductor

n-type semiconductor;

$$n >> p \Rightarrow J_T \cong nq\mu_n E = N_D q\mu_n E$$

where N_D is the shallow donor concentration

p-type semiconductor;

$$p >> n \Rightarrow J_T \cong pq\mu_p E = N_A q\mu_p E$$

where N_A is the shallow acceptor concentration

Variation of resistivity with temperature

Why does the *resistivity of a metal* <u>*increases*</u> with increasing temperature whereas the *resistivity of a semiconductor* <u>*decreases*</u> with increasing temperature?

$$\rho = \frac{1}{\sigma} = \frac{1}{nq\mu}$$

This fact is used in a real semiconductor device called a thermistor, which is used as a temperature sensing element.

The thermistor is a temperature – sensitive resistor; that is its terminal resistance is related to its body temperature. It has a negative temperature coefficient , indicating that its resistance will decrease with an increase in its body temperature.

Band Diagrams with Electric Field Present

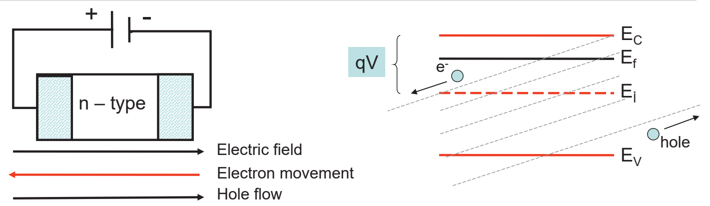
At equilibrium (with no external field)

All these energies are horizontal

E_C
E_i
E_V

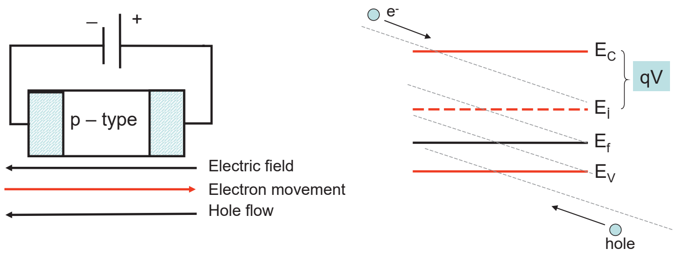
Pure/undoped semiconductor

How these energies will change with an applied field ?



Band Diagrams with Electric Field Present

- With an applied bias the band energies slope down for the given semiconductor. Electrons flow from left to right and holes flow from right to left to have their minimum energies for a p-type semiconductor biased as below.



Under drift conditions

- **Under drift conditions;** holes float and electrons sink. Since there is an applied voltage, currents are flowing and this current is called as drift current.

- There is a certain slope in energy diagrams and the depth of the slope is given by qV, where V is the battery voltage.

Work done on the charge carriers

Work = Force x distance = electrostatic force x distance

$$work = -qE \times L$$

$$= -q\frac{V}{L} \times L \Rightarrow work = -qV = \text{gain in energy}$$

$$\text{Slope of the band} = -\frac{qV}{L} = -qE = \text{Force on the electron}$$

$$E = \frac{V}{L}$$

where L is the length of the s/c.

Since there is a certain slope in the energies, i.e. the energies are not horizontal, the currents are able to flow.

Work done on the charge carriers

$$\text{Electrostatic Force} = -\text{gradiant of potential energy} = -\frac{dV}{dx}$$

$$-qE_x = -\frac{dE_i}{dx} \Rightarrow E_x = \frac{1}{q} \cdot \frac{dE_i}{dx} \qquad (1)$$

one can define electron's electrostatic potential as

$$E_x = -\frac{dV_n}{dx} \qquad (2)$$

comparison of equations (1) *and* (2) *gives,*

$$V_n = -\frac{E_i}{q}$$

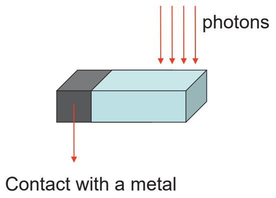
is a relation between V_n *and* E_i

Carrier Diffusion

Current mechanisms

Drift

Diffusion



$$P = nkT$$

$$\frac{dP}{dx} = \frac{dn}{dx}kT$$

$$\frac{dn}{dx} = \frac{1}{kT}\frac{dP}{dx}$$

Carrier Diffusion

➢Diffusion current is due to the movement of the carriers from high concentration region towards to low concentration region. As the carriers diffuse, a diffusion current flows. The force behind the diffusion current is the ***random thermal motion of carriers***.

$$\frac{dn}{dx} = \frac{1}{kT} \cdot \frac{dP}{dx}$$

➢ A concentration gradient produces a pressure gradient which produces the force on the charge carriers causing to move them.

How can we produce a concentration gradient in a semiconductor?

1) By making a semiconductor or metal contact.
2) By illuminating a portion of the semiconductor with light.

Illuminating a portion of the semiconductor with light

❖ By means of illumination, electron-hole pairs can be produced when the photon energy > E_g.

❖ So the increased number of electron-hole pairs move towards to the lower concentration region until they reach to their equilibrium values. So there is a number of charge carriers crossing per unit area per unit time, which is called as flux. Flux is proportional to the concentration gradient, dn/dx.

$$Flux = -D_n\frac{dn}{dx}$$

Flux

$$[Flux] = m^{-2} - s^{-1}$$

$$D = v_{th}l, \; [D] = m^2/s$$

The current densities for electrons and holes

$$J_n = -q\left(-D_n\frac{dn}{dx}\right) = qD_n\frac{dn}{dx} \quad \text{for electrons}$$

$$J_p = +q\left(-D_p\frac{dp}{dx}\right) = -qD_p\frac{dp}{dx} \quad \text{for holes}$$

$$[J_{n,p}] = [A/m^2]$$

Einstein Relation

Einstein relation relates the two independent current mechanism of mobility with diffusion;

$$\frac{D_n}{\mu_n} = \boxed{\frac{kT}{q}} \quad and \quad \frac{D_p}{\mu_p} = \boxed{\frac{kT}{q}} \quad for\ electrons\ and\ holes$$

Constant value at a fixed temperature

$$\frac{cm^2/\sec}{cm^2/V-\sec} = volt \qquad \frac{kT}{q} = \frac{(J/K)(K)}{C} = volt$$

$$\frac{kT}{q} = 25\ mV \quad at\ room\ temperature$$

Total Current Density

When both electric field (gradient of electric potential) and concentration gradient present, the total current density ;

$$J_n = q\mu_n nE + qD_n\frac{dn}{dx}$$

$$J_p = q\mu_p pE - qD_p\frac{dp}{dx}$$

$$J_{total} = J_n + J_p$$

Carrier Recombination and Diffusion Length

- By means of introducing excess carriers into an intrinsic s/c, the number of majority carriers hardly changes, but the number of minority carriers increases from a low- to high-value.
- When we illuminate our sample (n-type silicon with 10^{15} cm^{-3}) with light that produces 10^{14} cm^{-3} electron-hole pairs.
- The electron concentration (majority carriers) hardly changes, however hole concentration (minority carriers) goes from 1.96 x 10^5 to 10^{14} cm^{-3}.

Recombination Rate

- Minority carriers find themselves surrounded by very high concentration of majority carriers and will readily recombine with them.
- The recombination rate is proportional to excess carrier density, δ_p.

$$\frac{d\delta p}{dt} = -\frac{1}{\tau_p}\delta p \Rightarrow \delta p(t) = \delta p(0)\exp\left(-\frac{t}{\tau}\right)$$

Lifetime of holes

Excess hole concentration when t=0

Excess hole concentration decay exponentially with time.

Similarly, for electrons;

$$\frac{d\delta n}{dt} = -\frac{1}{\tau_n}\delta n \Rightarrow \delta n(t) = \delta n(0)\exp\left(-\frac{t}{\tau}\right)$$

Diffusion Length L

When excess carriers are generated in a specimen, the minority carriers diffuse a distance, a characteristic length, over which minority carriers can diffuse before recombining majority carriers. This is called as a diffusion length, L.

Excess minority carriers decay exponentially with diffusion distance.

$$\delta n(x) = \delta n(0)\exp\left(-\frac{x}{L_n}\right) \qquad \delta p(x) = \delta p(0)\exp\left(-\frac{x}{L_p}\right)$$

Excess electron concentration when x=0

Diffusion length for electrons

$$L_n = \sqrt{D_n\tau_n}$$

Diffusion length for holes

$$L_p = \sqrt{D_p\tau_p}$$

Light Sources

LED – Light Emitting Diode

Solid State Lighting (SSL)

Solid-state lighting (SSL) is a type of lighting that uses semiconductor light-emitting diodes (LEDs). Organic light-emitting diodes (OLED), or polymer light-emitting diodes (PLED) as sources of illumination rather than electrical filaments, plasma (used in arc lamps such as fluorescent lamps), or gas.

Advantages of LED Lighting

- **Lifespan** – Lifespan can exceed 50,000 Hours as compared to 1,000 Hours for filament bulbs
- **Robustness** – No moving parts, No filaments, No glass
- **Size** – compact package typically 5 mm in diameter
- **Energy Efficiency** – 30-70% less energy than other light sources.
- **Environmental Safety** – No Mercury or Lead. Easy to dispose
- **Versatility** – available in a variety of colours and can be tuned
- **Cool** – Less radiant heat than HID or Incandescent types
- **Luminous Efficacy** – LED can produce up to 300 lumens/Watt

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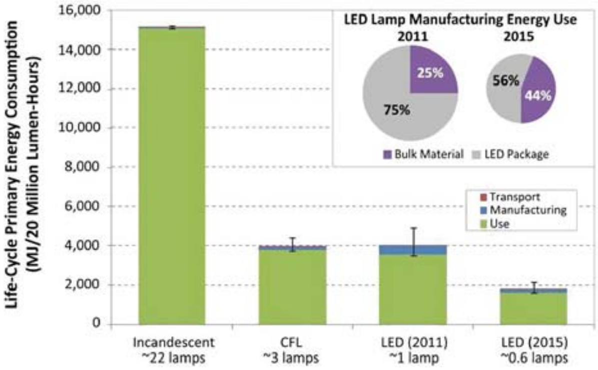
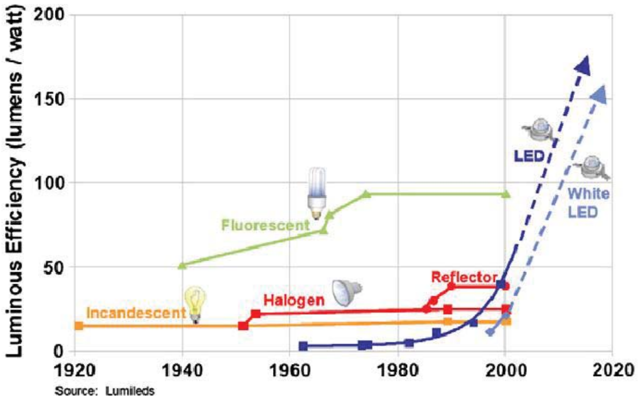
Energy Usage Comparison



Luminous Performance Evolution

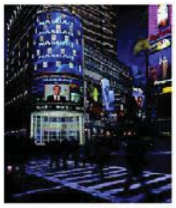


Source: Lumileds

Global LED Market approx. 90 B$ in 2024

Cellphone (Nokia)

Traffic signals (Gelcore)



Large Displays (NASDAQ)

streetlights

TVs (LED DLPtm) (samsung)

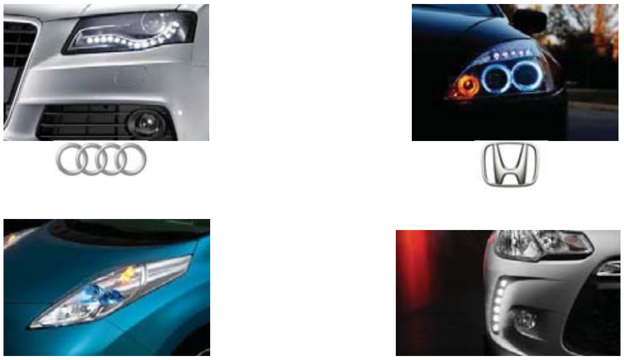
Automotive

LED in Architectural Lighting


Aspire Tower, Doha, Qatar


Takarazuka University, Japan

Car Headlights



Portable Desk/Task Lighting


6 Watt LED Desk Lamp


Halley LED Desk Lamp

Indoor market 3 times larger than Outdoor

Air/Water Purification System

- Fruit and Vegetable Storage Life Extended 1 week
- Water Purification: UV LED to kill bacteria


Mitsubishi Refrigerator MR-W55H,
UV LED 375 nm, 590 nm, Blue LED


UV Water Purifier
(Credit: Hydro-Photon Inc.)

Costs associated with various white lighting

•**Incandescent**
Traditional bulbs that are inefficient, wasting 90% of their energy as heat. They emit about 15 lumens per watt and have a lifespan of around 1,200 hours. Cost: $1

•**Halogen**
A type of incandescent lighting that's more efficient than traditional incandescent bulbs, but less efficient than LED and CFL bulbs. They emit about 25 lumens per watt and have a lifespan of around 2,500 hours. Cost: $15

•**HID**
High-intensity discharge bulbs that are brighter and more efficient than halogens. They are used as running lights on vehicles. They have a lifespan of up to 15,000 hours. Cost: $40

•**CFL**
Compact fluorescent lamps that are more energy efficient than incandescent bulbs. They emit 60 lumens per watt and have a lifespan of around 10,000 to 15,000 hours. Cost: $30

•**LED**
Light emitting diodes that are the most energy efficient of all the light bulbs. They emit around 75 lumens per watt and can last up to 50,000 hours. Cost: $5

Luminous Efficacy

- Luminous efficacy is a property of light sources, which indicates what portion of the emitted electromagnetic radiation is usable for human vision. **It is the ratio of emitted luminous flux to radiant flux.** Luminous efficacy is related to the overall efficiency of a light source for illumination, but the overall lighting efficiency also depends on how much of the input energy is converted into electromagnetic waves (whether visible or not).
- •In SI, luminous efficacy has units of lumens per watt (lm/W). **Photopic** luminous efficacy (daylight) has a maximum possible value of 683 lm/W, for the case of monochromatic light at a wavelength of 555nm (green). **Scotopic** luminous efficacy (night) reaches a maximum of 1700 lm/W for narrowband light of wavelength 507 nm.

Luminous Efficiency Comparison

Type	Luminous efficacy (Lm/W)	Luminous Efficiency
Tungsten light bulb, typical, 2800 K	15	2%
Class M star (Antares, Betelgeuse), 3300 K	30	4%
Black body, 4000 K, ideal	47.5	7%
Class G star (Sun, Capella), 5800 K	93	13.60%
Black body, 7000 K, ideal	95	14%
Ideal monochromatic source: 555 nm (in air)	683	100%

Flux

- The lumen (lm) is the SI unit of luminous flux, a measure of the perceived power of light is equal to the amount of light emitted per second in a unit solid angle of one steradian from a light source of 1 candela.
- Luminous flux differs from radiant flux, the measure of the total power of light emitted, in that luminous flux is adjusted to reflect the varying sensitivity of the human eye to different wavelengths of light

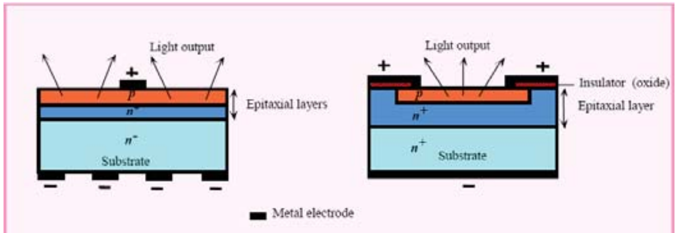
LED Principles

- If forward bias V is applied › voltage drop mainly occurs across the depletion region › Built-in potential Vo is reduced to Vo–V. › Allow the electrons from n+ side to diffuse (or become injected), into the p-side
- The hole injection component from p- into n+ side is much smaller.
- The recombination of injected electrons in the depletion region as well as in the neutral p- side results in photons emission.
- Recombination mainly occurs inside depletion region and within a volume extending over the diffusion length Le of the electrons in the p- side. Recombination zone is called active region.
- The phenomenon of light emission from EH pair recombination as result of minority carrier injection is called **injection electroluminescense**.

LED Structure

- The p- side is on the surface from which light is emitted as is therefore made narrow (a few microns) to allow the photon to escape without being reabsorbed.
- The n- side is heavily doped to ensure that the most of the recombination takes place in the p- side.
- The photons which are emitted toward the n- side become either absorbed or reflected back at the substrate interface depending on the substrate thickness and the exact structure of the LED.

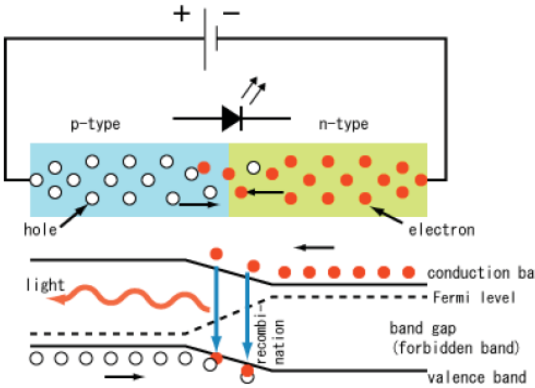
Illustration of typical planar surface emitting LED devices



p-layer grown epitaxially on an *n+* substrate

First *n+* is epitaxially grown and then *p* region is formed by dopant diffusion into the epitaxial layer

Inner Workings of a LED



Radiative vs. Non-radiative Recombination

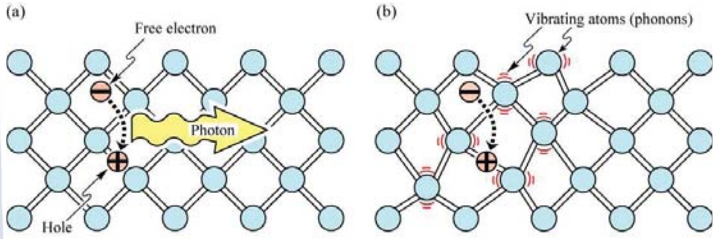


Fig. 2.5. (a) Radiative recombination of an electron-hole pair accompanied by the emission of a photon with energy $h\nu \approx E_g$. (b) In non-radiative recombination events, the energy released during the electron-hole recombination is converted to phonons (adopted from Shockley, 1950).

Recombination Mechanisms

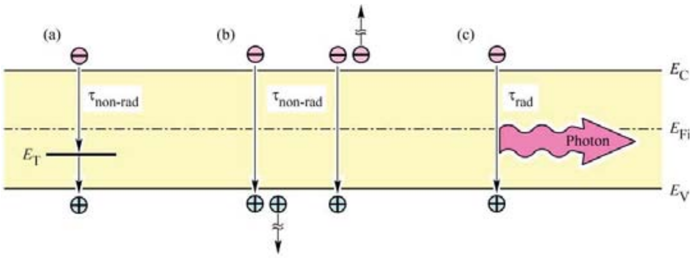
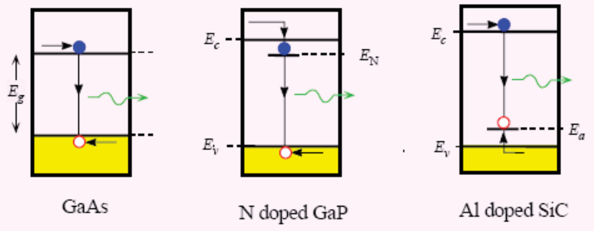


Fig. 2.6. Band diagram illustrating non-radiative recombination: (a) via a deep level, (b) via an Auger process and (c) radiative recombination.

Radiative Recombination



GaAs

N doped GaP

Al doped SiC

Photon emission in a direct bandgap semiconductor

GaP is an indirect bandgap semiconductor. When doped with nitrogen there is an electron trap at E_N. Direct recombination between a trapped electron at E_N and a hole emits a photon.

In *Al* doped *SiC*, EHP recombination is through an acceptor level like E_a.

LED Electrical Basics

Shockley equation for p-n junction diodes

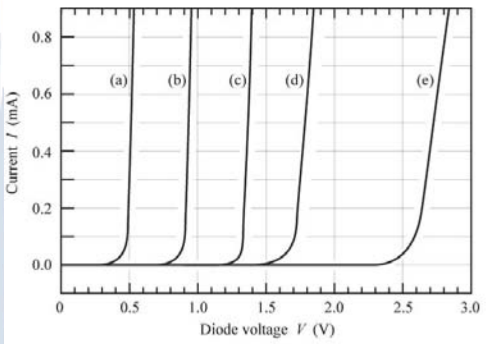
$$I = eA\left(\sqrt{\frac{D_p}{\tau_p}}\,p_{n0} + \sqrt{\frac{D_n}{\tau_n}}\,n_{p0}\right)\left(e^{eV/kT} - 1\right)$$

$$= eA\left(\sqrt{\frac{D_p}{\tau_p}}\,\frac{n_i^2}{N_D} + \sqrt{\frac{D_n}{\tau_n}}\,\frac{n_i^2}{N_A}\right)\left(e^{eV/kT} - 1\right)$$

$$= I_s\left(e^{eV/kT} - 1\right)$$

where I_s is the saturation current

Diode IV Characteristics

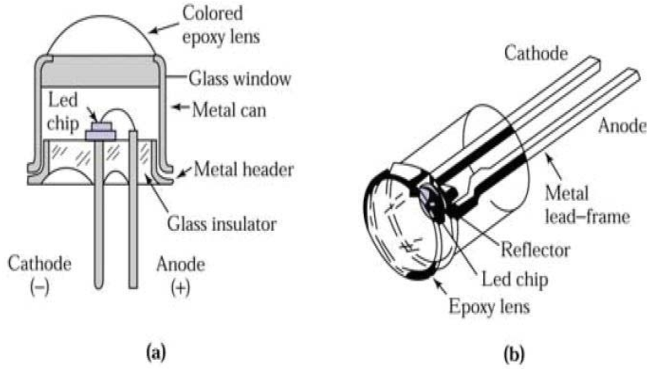


$T = 295$ K

(a) Ge $E_g \approx 0.7$ eV
(b) Si $E_g \approx 1.1$ eV
(c) GaAs $E_g \approx 1.4$ eV
(d) GaAsP $E_g \approx 2.0$ eV
(e) GaInN $E_g \approx 2.9$ eV

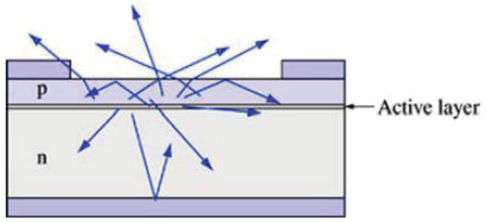
Fig. 4.2. Room-temperature current–voltage characteristics of p-n junctions made from different semiconductors.

LED Structure

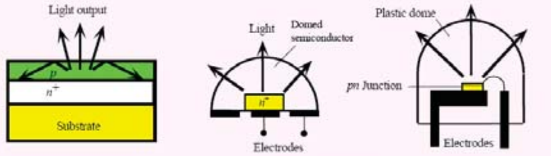


LED Structure

- ❑ A generic surface-emitting LED.
- ➢ Some photons are lost by reabsorption in the bulk, Fresnel reflection from the surface, and total internal reflection.



LED Structure



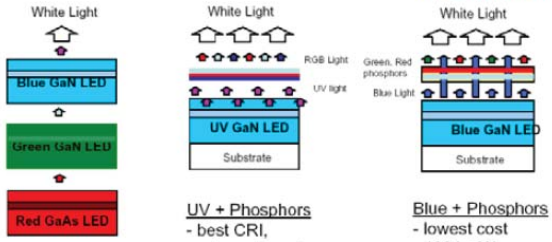
Some light suffers total internal reflection and cannot escape

Internal reflections can be reduced and hence more light can be collected by shaping the semiconductor into a dome so that the angles of incidence at the semiconductor-air surface are smaller than the critical angle

An economic method of allowing more light to escape from the LED is to encapsulate it in a transparent plastic dome

Methods for Generating white light

100 lm/W (2005)



Multi-Chip, RGB
- good efficiency
- highest cost
- tunable color

UV + Phosphors
- best CRI,
- color uniformity
- low cost
- improve reliability

Blue + Phosphors
- lowest cost
- 100 lm/W
- 90% market share

LED Materials

- There are various direct bandgap semiconductor materials that can be readily doped to make commercial pn juction LEDs that emit radiation in the red and infrared range of wavelengths.
- Class of commercial semiconductor materials that cover visible spectrum is the **III-V ternary alloys** (three elements) based on alloying GaAs and GaP, which are denoted as $GaAs_{1-y}P_y$.
- When y < 0.45, this alloy is a direct bandgap semiconductor and hence the EHP recombination process is direct (fig. a).
- The emitted wavelengths are 630 nm (y = 0.45) – 870 nm (y = 0).
- For indirect bandgap, y > 0.45, EHP recombination processes occur through recombination centers and invlve lattice vibrations (phonon).
- **Isoelectronic impurities**, such as nitrogen (in the same group V as P) → some N atoms substitute for P atoms.

LED Materials

- Positive nucleus of N is less shielded → conduction electron in the neighborhood of an N atom will be attracted and may traped at this site. → N atom then have localized energy levels, or electron traps, E_N, near the conduction band (fig b).
- The trapped electron then can attract a hole in its vicinity → recombine with it and emit a photon.
- The emitted photon energy is slightly less than E_g.
- The recombination process depends on N doping → less efficient.
- Mainly used from green, yellow, and orange LEDs.
- Two types of blue LED materials: GaN alloy (InGaN) and Al doped SiC.
- The localized energy level captures a hole from the valence band and a conduction electron then recombines with this hole to emit a photon (fig c).
- More efficient blue LEDs using direct bandgap compound II-VI semiconductors, such as ZnSe.

LED Materials

	Color	Wavelength [nm]	Semiconductor material
⬛	Infrared	$\lambda > 760$	Gallium arsenide (GaAs) Aluminium gallium arsenide (AlGaAs)
🟥	Red	$610 < \lambda < 760$	Aluminium gallium arsenide (AlGaAs) Gallium arsenide phosphide (GaAsP) Aluminium gallium indium phosphide (AlGaInP) Gallium(III) phosphide (GaP)
🟧	Orange	$590 < \lambda < 610$	Gallium arsenide phosphide (GaAsP) Aluminium gallium indium phosphide (AlGaInP) Gallium(III) phosphide (GaP)
🟨	Yellow	$570 < \lambda < 590$	Gallium arsenide phosphide (GaAsP) Aluminium gallium indium phosphide (AlGaInP) Gallium(III) phosphide (GaP)
🟩	Green	$500 < \lambda < 570$	**Traditional green:** Gallium(III) phosphide (GaP) Aluminium gallium indium phosphide (AlGaInP) Aluminium gallium phosphide (AlGaP) **Pure green:** Indium gallium nitride (InGaN) / Gallium(III) nitride (GaN)
🟦	Blue	$450 < \lambda < 500$	Zinc selenide (ZnSe) Indium gallium nitride (InGaN) Silicon carbide (SiC) as substrate Silicon (Si) as substrate—under development
🟪	Violet	$400 < \lambda < 450$	Indium gallium nitride (InGaN)
🟪	Purple	multiple types	Dual blue/red LEDs, blue with red phosphor, or white with purple plastic
⬛	Ultraviolet	$\lambda < 400$	Diamond (235 nm) Boron nitride (215 nm) Aluminium nitride (AlN) (210 nm) Aluminium gallium nitride (AlGaN) Aluminium gallium indium nitride (AlGaInN)—down to 210 nm
🟪	Pink	multiple types	Blue with one or two phosphor layers: yellow with red, orange or pink phosphor added afterwards, or white with pink pigment or dye.
	White	Broad spectrum	Blue/UV diode with yellow phosphor

LED Materials

- Other various commercial direct bandgap semiconductor material that emit the red and infrared wavelengths: ternary, quartenary alloys based on III and V elements.
- Emitted radiation ranges from 640 – 870 nm (from deep red light to infrared).
- External efficiency η_{external} of an LED: efficiency of conversion of electrical energy into an emitted external external energy.

$$\eta_{\text{external}} = \frac{P_{\text{out}}(\text{Optical})}{IV} \times 100\%$$

- The input power: product of diode current and voltage.

LED External Conversion Efficiency

The external power or conversion efficiency η_{ext} is defined as

$$\eta_{\text{ext}} = \frac{\text{Optical power output}}{\text{Electrical power input}} = \frac{P_o}{IV}$$

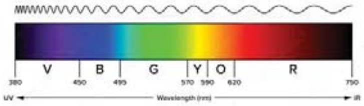
One of the major factors reducing the external power efficiency is the loss of photons in extracting the emitted photons which suffer reabsorption in the pn junction materials, absorption outside the semiconductors and various reflections at interfaces.
The total light output power from a particular AlGaAs red LED is 2.5 mW when the current is 50 mA and the voltage is 1.6 V.
Calculate its external conversion efficiency.

Solution

$$\eta_{\text{ext}} = \frac{P_o}{IV} = \frac{2.5\times10^{-3}\ \text{W}}{(50\times10^{-3}\ \text{A})(1.6\ \text{V})} = 0.03125 = \mathbf{3.125\ \%}$$

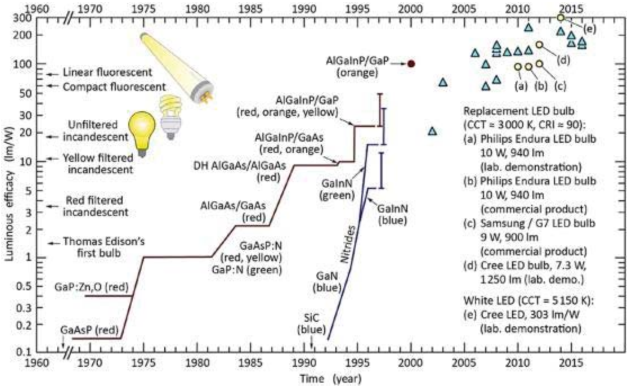
LED Materials and Colour

Materials	Wavelength	Colour
GaInN	370	🟪
GaInN	450	🟦
GaInN	525	🟩
InGaAlP/GaAs	562	🟩
InGaAlP/GaAs	574	🟩
InGaAlP/GaP	590	🟨
InGaAlP	644	🟥
GaAlAs/GaAs	660	🟥
GaP	700	🟥
GaAlAs	880	IR
GaAs	940	IR



Color	Wavelength (nm)	Frequency (THz)	Photon Energy (eV)
🟪 Violet	380 – 450	670 – 790	2.75 – 3.26
🟦 Blue	450 – 485	620 – 670	2.56 – 2.75
🟦 Cyan	485 – 500	600 – 620	2.48 – 2.56
🟩 Green	500 – 565	530 – 600	2.19 – 2.48
🟨 Yellow	565 – 590	510 – 530	2.10 – 2.19
🟧 Orange	590 – 625	480 – 510	1.98 – 2.10
🟥 Red	625 – 750	400 – 480	1.65 – 1.98

Historical Development of LEDs



Quaternary alloy Example

InGaAsP on InP substrate

The quaternary alloy $\mathbf{In_{1-x}Ga_xAs_{1-y}P_y}$ grown on an InP crystal substrate is a suitable commercial semiconductor material for infrared LED and laser diode applications. The device requires that the InGaAsP layer is lattice matched to the InP crystal substrate to avoid crystal defects in the InGaAsP layer. This in turn requires that $\mathbf{y \approx 2.2x}$. The bandgap energy E_g of the alloy in eV is given by the empirical relationship:

$$\mathbf{E_g = 1.35 - 0.72y + 0.12y^2 \qquad ; 0 \le x \le 0.47}$$

Calculate the composition of InGaAsP alloy for peak emission wavelength of **1.3 μm**.

($h = 6.626x10^{-34}$ Js, $K_BT = 0.0259$ eV at 300K, $c = 3x10^8$ m/s, $q = 1.6x10^{-19}$ C)

The photon energy at peak emission is: $\frac{hc}{\lambda} = E_g + K_BT$ in Joules

Rewriting equation in eV: $E_g = \frac{hc}{\lambda q} - 0.0259$ where $\lambda = 1.3X10^{-6}$ m, T = 300K

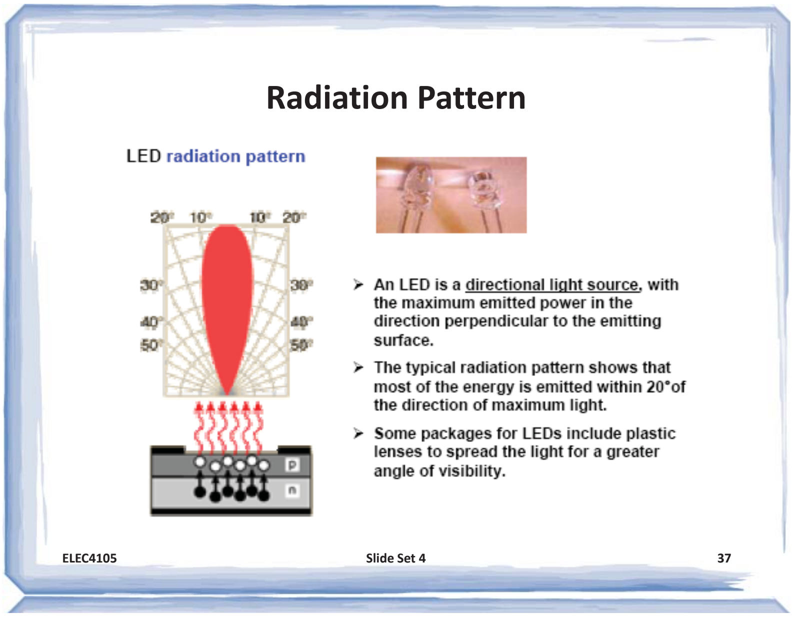
$$E_g = \frac{6.626\ x\ 10^{-34}\ .\ 3x10^8}{1.3x10^{-6}.\ 1.6x10^{-19}} - 0.0259 = 0.928\ eV$$

The InGaAsP alloy must therefore satisfy: $0.928 = 1.35 - 0.72y + 0.12y^2$

$$0.12y^2 - 0.72y + 0.422 = 0$$

Solving: y = **0.66** and x = 0.66/2.2 = **0.3**

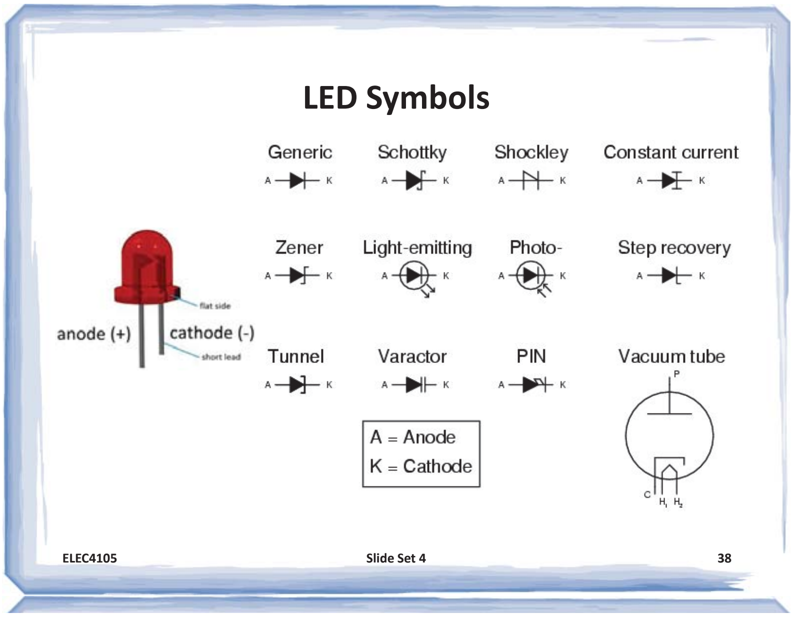
The quaternary alloy composition is $\mathbf{In_{0.7}Ga_{0.3}As_{0.34}P_{0.66}}$



Radiation Pattern

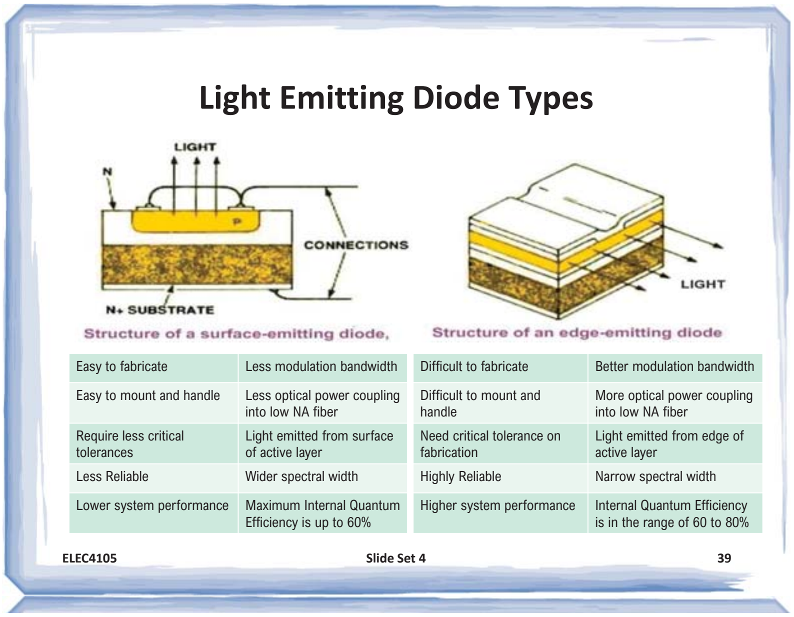
LED **radiation pattern**

- An LED is a directional light source, with the maximum emitted power in the direction perpendicular to the emitting surface.
- The typical radiation pattern shows that most of the energy is emitted within 20°of the direction of maximum light.
- Some packages for LEDs include plastic lenses to spread the light for a greater angle of visibility.



LED Symbols

A = Anode
K = Cathode

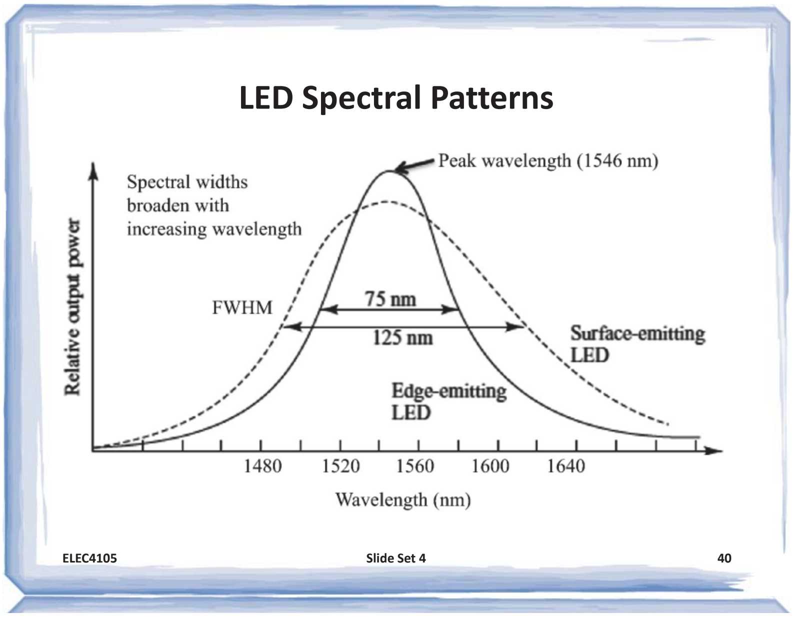


Light Emitting Diode Types

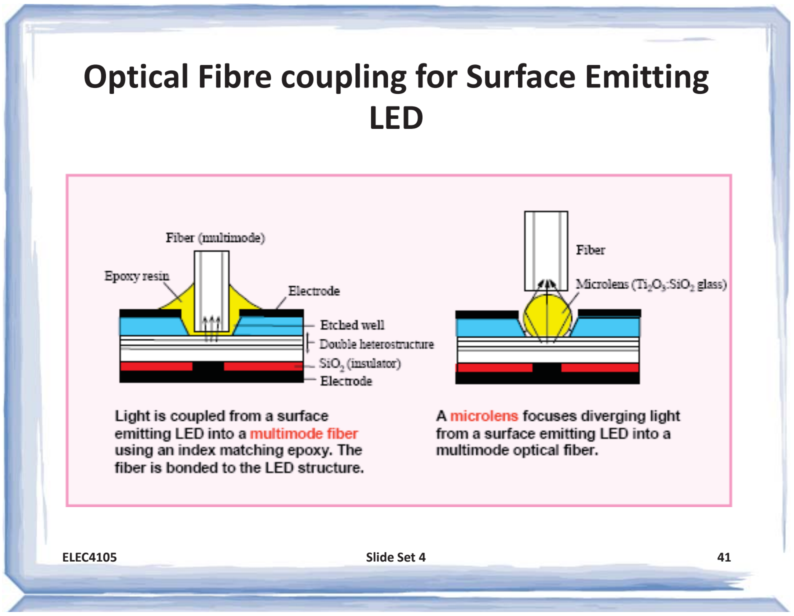
Structure of a surface-emitting diode,

Structure of an edge-emitting diode

Easy to fabricate	Less modulation bandwidth	Difficult to fabricate	Better modulation bandwidth
Easy to mount and handle	Less optical power coupling into low NA fiber	Difficult to mount and handle	More optical power coupling into low NA fiber
Require less critical tolerances	Light emitted from surface of active layer	Need critical tolerance on fabrication	Light emitted from edge of active layer
Less Reliable	Wider spectral width	Highly Reliable	Narrow spectral width
Lower system performance	Maximum Internal Quantum Efficiency is up to 60%	Higher system performance	Internal Quantum Efficiency is in the range of 60 to 80%



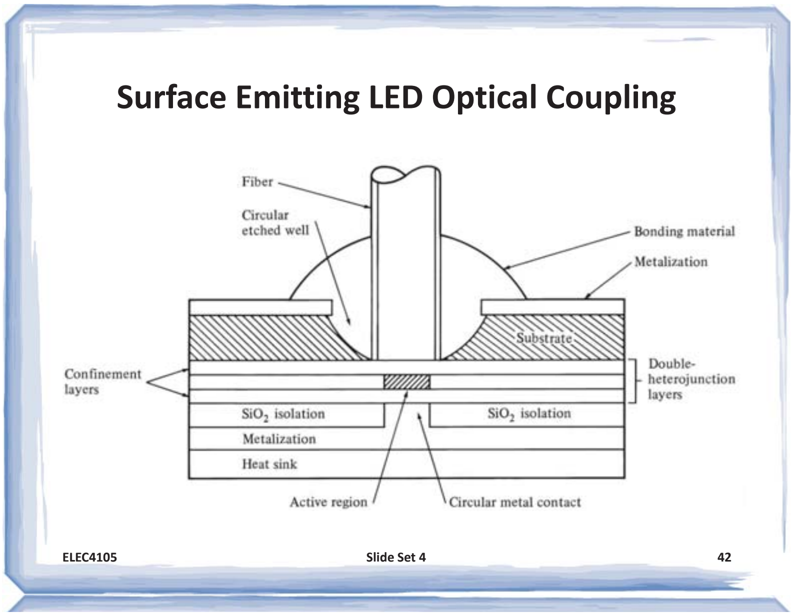
LED Spectral Patterns



Optical Fibre coupling for Surface Emitting LED

Light is coupled from a surface emitting LED into a multimode fiber using an index matching epoxy. The fiber is bonded to the LED structure.

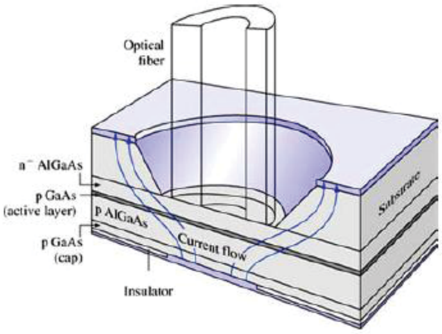
A microlens focuses diverging light from a surface emitting LED into a multimode optical fiber.



Surface Emitting LED Optical Coupling

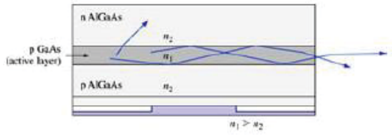
Burrus Type LED

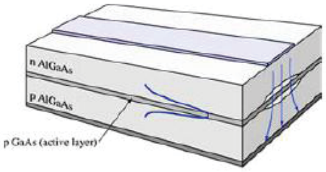
A **Burrus-type LED**. This one uses a Double Heterostructure to confine the carriers, making recombination more efficient. The etched opening in the LED helps align and couple an optical fiber.



Edge Emitting LED

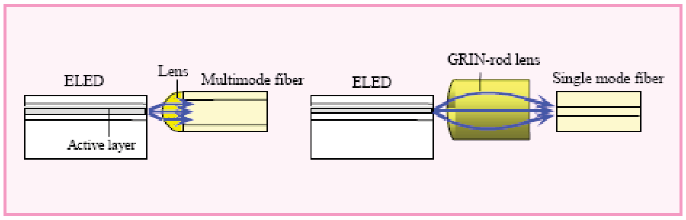
- ☐ In an **Edge-emitting LED**, the higher-index active layer acts as a waveguide for photons traveling at less than the critical angle.





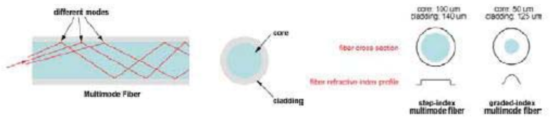
Optical Fibre Coupling in Edge Emitted LED

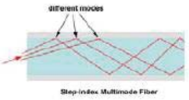
- ☐ Light from an **Edge emitting LED** is coupled into a fiber typically by using a lens or a GRIN rod lens

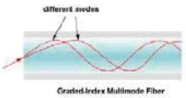


GRIN = Gradient Index

Type of Multimode Fibre

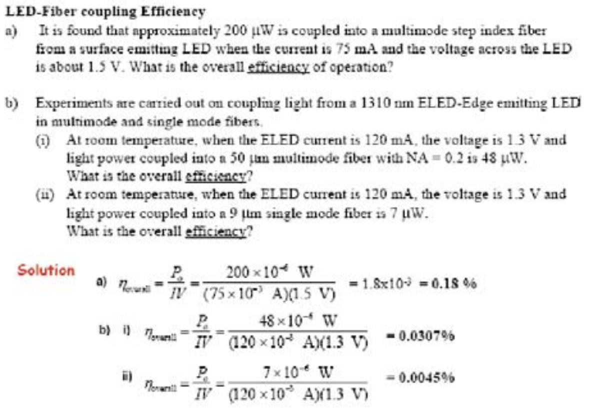






Step-index multimode fibers are mostly used for imaging and illumination. Graded-index multimode fibers are used for data communications and networks carrying signals moderate distances - typically no more than a couple of kilometers

Fibre Coupling Efficiency



LED-Fiber coupling Efficiency

a) It is found that approximately 200 µW is coupled into a multimode step index fiber from a surface emitting LED when the current is 75 mA and the voltage across the LED is about 1.5 V. What is the overall efficiency of operation?

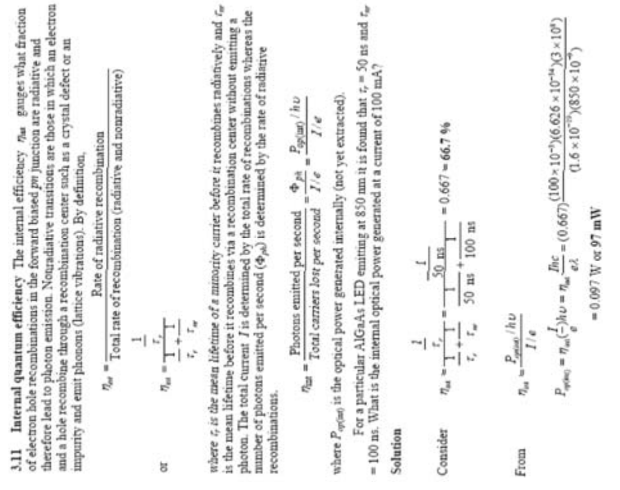
b) Experiments are carried out on coupling light from a 1310 nm ELED-Edge emitting LED in multimode and single mode fibers.
   (i) At room temperature, when the ELED current is 120 mA, the voltage is 1.3 V and light power coupled into a 50 µm multimode fiber with NA = 0.2 is 48 µW. What is the overall efficiency?
   (ii) At room temperature, when the ELED current is 120 mA, the voltage is 1.3 V and light power coupled into a 9 µm single mode fiber is 7 µW. What is the overall efficiency?

Solution

a) $$\eta_{overall} = \frac{P_o}{IV} = \frac{200\times10^{-6}\ \text{W}}{(75\times10^{-3}\ \text{A})(1.5\ \text{V})} = 1.8\text{x}10^{-3} = \mathbf{0.18\ \%}$$

b) i) $$\eta_{overall} = \frac{P_o}{IV} = \frac{48\times10^{-6}\ \text{W}}{(120\times10^{-3}\ \text{A})(1.3\ \text{V})} = \mathbf{0.0307\%}$$

ii) $$\eta_{overall} = \frac{P_o}{IV} = \frac{7\times10^{-6}\ \text{W}}{(120\times10^{-3}\ \text{A})(1.3\ \text{V})} = \mathbf{0.0045\%}$$



3.11 Internal quantum efficiency The internal efficiency η_{int} gauges what fraction of electron hole recombinations in the forward biased *pn* junction are radiative and therefore lead to photon emission. Nonradiative transitions are those in which an electron and a hole recombine through a recombination center such as a crystal defect or an impurity and emit phonons (lattice vibrations). By definition,

$$\eta_{int} = \frac{\text{Rate of radiative recombination}}{\text{Total rate of recombination (radiative and nonradiative)}}$$

or $$\eta_{int} = \frac{\frac{1}{\tau_r}}{\frac{1}{\tau_r} + \frac{1}{\tau_{nr}}}$$

where *τ_r is the mean lifetime of a minority carrier before it* recombines radiatively and τ_{nr} is the mean lifetime before it recombines via a recombination center without emitting a photon. The total current I is determined by the total rate of recombinations whereas the number of photons emitted per second (Φ_{ph}) is determined by the rate of radiative recombinations.

$$\eta_{int} = \frac{\text{Photons emitted per second}}{\textit{Total carriers lost per second}} = \frac{\Phi_{ph}}{I/e} = \frac{P_{op(int)}/h\upsilon}{I/e}$$

where $P_{op(int)}$ is the optical power generated internally (not yet extracted).

For a particular AlGaAs LED emitting at 850 nm it is found that $\tau_r = 50$ ns and $\tau_{nr} = 100$ ns. What is the internal optical power generated at a current of 100 mA?

Solution

Consider $$\eta_{int} = \frac{\frac{1}{\tau_r}}{\frac{1}{\tau_r} + \frac{1}{\tau_{nr}}} = \frac{\frac{1}{50\ \text{ns}}}{\frac{1}{50\ \text{ns}} + \frac{1}{100\ \text{ns}}} = 0.667 = \mathbf{66.7\ \%}$$

From $$\eta_{int} = \frac{P_{op(int)}/h\upsilon}{I/e}$$

$$P_{op(int)} = \eta_{int}\left(\frac{I}{e}\right)h\upsilon = \eta_{int}\frac{Ihc}{e\lambda} = (0.667)\frac{(100\times10^{-3})(6.626\times10^{-34})(3\times10^{8})}{(1.6\times10^{-19})(850\times10^{-9})}$$

$$= 0.097\ \text{W or } \mathbf{97\ mW}$$

Light Sources

LASER

Contents

- Lasers
  - Basic Principles
  - Applications
- Gas Lasers
- Semiconductor Lasers
- Semiconductor Lasers in Optical Networks
- Improvement in Basic Design
- Recent Advances

Lasers: Basic Principle

Light Amplification by Stimulated Emission of Radiation

Key Terms:

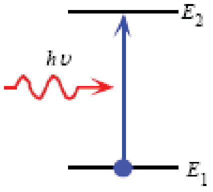
- Stimulated Emission
- Metastable State
- Population Inversion

Properties of LASERS

- **Monochromaticity**
- **Coherence**
- **Beam Divergence**
- **High Irradiance**
- **Properties vary with type of Lasers:**
  - **Gas, Solid, Semiconductor**

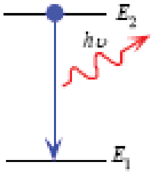
Spontaneous vs. Stimulated Emission

An electron in an atom can be excited from one energy level E1 to a higher energy level E2 by absorption → photon absorption **hv = E2 – E1**. Note: v is lowercase nu representing frequency.



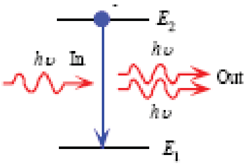
Spontaneous vs. Stimulated Emission

- Two possibilities of emission (an electron moves/transits down in energy to an unoccupied energy level -> emits a photon)
  - Spontaneous
  - Induced (Stimulated)
- Spontaneous emission: random direction -> random photon.
- Transition for E2 to E1 as if the electron is oscillating with a frequency *v*



Spontaneous vs. Stimulated Emission

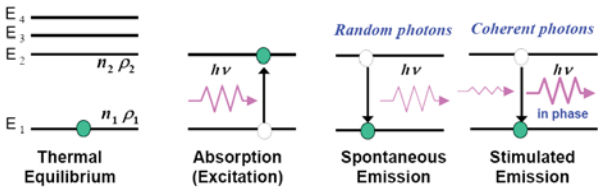
- Stimulated emission: incoming photon of energy h*v* = E2– E1 stimulates the whole emission process by inducing the electron at E2 to transit down to E1.
- Emitted photon: in phase, same direction, same polarization, same energy with incoming photon -> two outgoing photons.
- To obtain stimulated emission → the incoming photon should not be absorbed by another atom at E1.



Spontaneous vs. Stimulated Emission

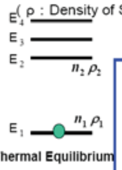
- Although we consider transitions of an electron in an atom, we could have just well described photon absorption, spontaneous and stimulated emission in term of energy transitions of the atom itself in which case E1 and E2 represent the energy levels of the atom.
- Consider the collection of atoms to amplify light → we must have the majority of atoms at the energy level E2. Otherwise the incoming photon will be absorbed by the atom at E1.
- **Population inversion**: more atoms at E2 than at E1.
- In steady state incoming photon will cause as many upward excitations as downward stimulated emissions → for only two energy levels → we can never achieve atom population at E2 greater than E1.

Lasers: Basic Principle



Lasers: Basic Principle

Absorption and Radiation Processes



Boltzmann Distribution

Number of electron at E1 $n_1 = \rho_1 f(E_1) = \rho_1 e^{[-(E_1-E_f)/kT]}$

Number of electron at E2 $n_2 = \rho_2 f(E_2) = \rho_2 e^{[-(E_2-E_f)/kT]}$

$$\frac{f(E_2)}{f(E_1)} = \frac{n_2\,\rho_1}{n_1\,\rho_2} = e^{[-(E_2-E_1)/kT]} \rightarrow \left(\frac{n_2}{\rho_2}\right) = \left(\frac{n_1}{\rho_1}\right) e^{[-(E_2-E_1)/kT]}$$

- ➢ $(E_2 - E_1) > 0 \rightarrow \left(\frac{n_2}{\rho_2}\right) < \left(\frac{n_1}{\rho_1}\right)$
- ➢ If $\left(\frac{n_2}{\rho_2}\right) > \left(\frac{n_1}{\rho_1}\right)$ can be achieved by **pumping**,

→ $kT < 0$ (negative temperature: *Population inversion*)

Lasers: Basic Principle

‘'Negative’ Temperature concept

Question: What negative temperature would describe a system in which the population of the upper energy level exceeds that of the lower level by 10% and the energy difference between the two levels is 2.26 eV?

Answer: Number of atoms in a excited state $N_2 = N_0 e^{-(E_2-E_0)/kT}$.

$\therefore \frac{N_2}{N_0} = e^{-(E_2-E_0)/kT}$.

$\ln\left(\frac{N_2}{N_0}\right) = -(E_2 - E_0)/kT$

or, $-T = \frac{E_2 - E_0}{k \ln\left(\frac{N_2}{N_0}\right)}$

k = Boltzman constant

T = Temperature.

N_2 = Number of atoms in a excited state.

Now,

$E_2 - E_0 = 2.2\,eV$ & $\frac{N_2}{N_0} = 1.10$, k = 8.62

therefore,

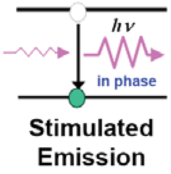
$-T = \frac{2.2\,eV}{(8.62 \times 10^{-5}\,eV/K)\ln(1.10)}$

$\boxed{T = -2.75 \times 10^5\ K}$

Answer:- Negative temperature of 2.75×10⁵ K or 2K will describe a system.

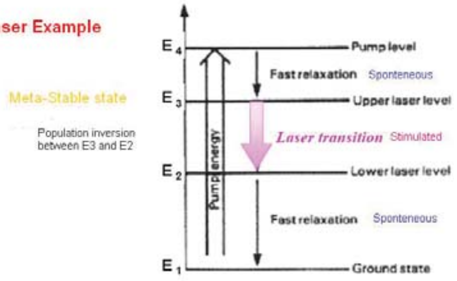
Lasers: Basic Principle

Absorption and Radiation Processes



Coherent photons
In phase
Same energy
Same direction
Same polarization

Laser: Basic Principle



Basic requirement for Lasing action:
Metastable state, Population inversion, Optical resonant cavity

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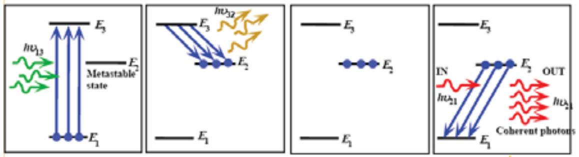
Laser: Basic Principle

Lasing Action

3-Level Laser Example



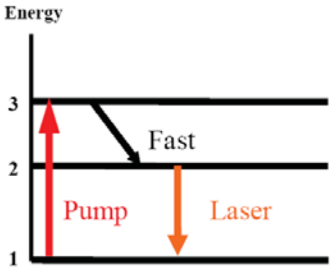
Atoms in the ground state are pumped up to the energy level E_3 by incoming photons of energy $h\nu_{13} = E_3 - E_1$.

Atoms at E_3 rapidly decay to the metastable state at energy level E_2 by emitting photons or emitting lattice vibrations; $h\nu_{32} = E_3 - E_2$

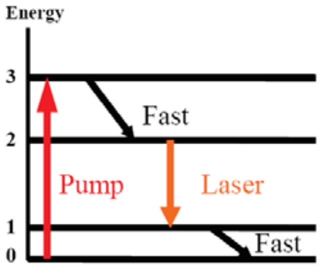
As the states at E_2 are long-lived, they quickly become populated and there is a population inversion between E_2 and E_1.

A random photon (from a spontaneous decay) of energy $h\nu_{21} = E_2 - E_1$ can initiate stimulated emission. Photons from this stimulated emission can themselves further stimulate emissions leading to an avalanche of stimulated emissions and coherent photons being emitted.

Laser: Basic Principle



3 Energy levels laser



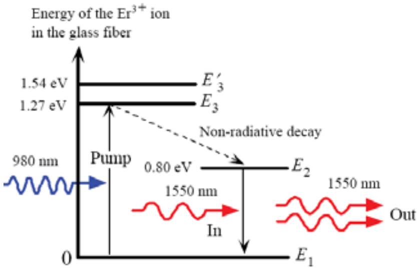
4 Energy levels laser

Optical Fibre Amplifier

- A light signal traveling long distances will suffer attenuation. → It is necessary to regenerate the light signal at certain intervals for long haul communications over several thousand miles.
- Practical optical amplifier is based on the **erbium ion (Er3+) doped fiber amplifier** (EDFA).
- The core region of an optical fiber is doped with Er3+ or with neodymium ion (Nd3+).
- The host fiber material is a glass based on SiO_3-GeO_2 or Al_2O_3. → Easily fused to a single mode long distance optical fiber by technique called splicing.

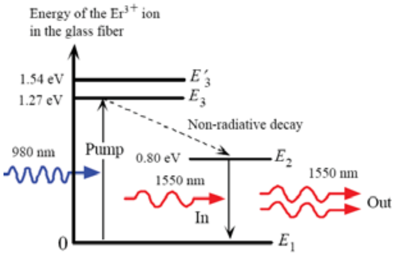
Optical Fibre Amplifier

- Er3+ has energy level as indicated in the figure below.
- Er3+ is optically pumped from laser diode to excite them to E3.
- The Er3+ ions decay rapidly from E3 to E2 (**long-lived**) energy level ~ 10 ms.
- The decay from E3 to E2 involves energy losses by radiation-less transition (phonon emission).



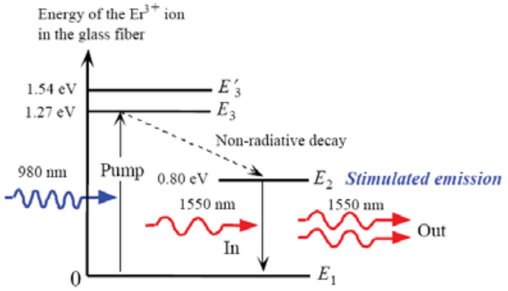
Optical Fibre Amplifier

- The accumulated Er^{3+} ions at E2 leads to a population inversion between E2 and E1.
- Signal photons at 1550 nm have energy of 0.80 eV (E2 – E1), and give rise to stimulated transitions of Er^{3+} ions from E2 to E1.
- Meanwhile, any Er^{3+} ions left at E1will absorb in incoming 1550 nm photons to reach E2.



Optical Fibre Amplifier

EDFA: Er-doped Optical Amplifier



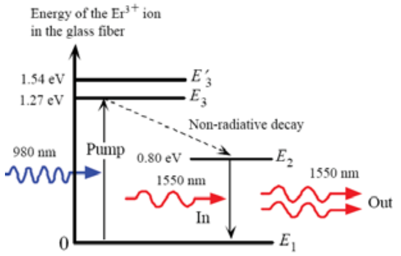
Energy diagram for the Er^{3+} ion in the glass fiber medium and light amplification by stimulated emission from E_2 to E_1. Dashed arrows indicate radiationless transitions (energy emission by lattice vibrations)

Optical Fibre Amplifier

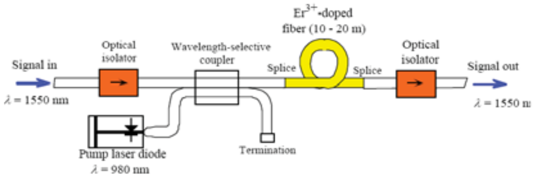
- Thus, to achieve light amplification we must have stimulated emission exceeding absorption.
- Only possible if more Er3+ ions at E2 (N2) than at E1 (N1).

- The net optical gain G_{op}: where K is a constant which depends on the pumping intensity

$$G_{op} = K(N_2 - N_1)$$



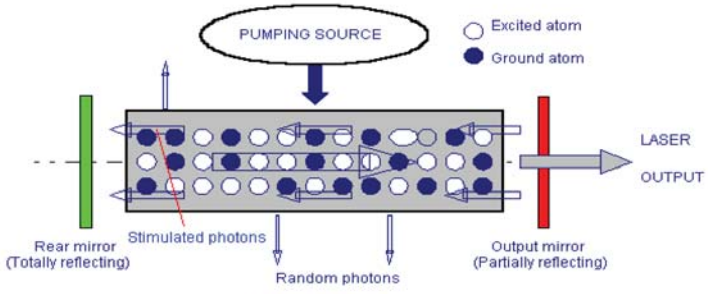
Optical Fibre Amplifier



A simplified schematic illustration of an EDFA (optical amplifier). The erbium-ion doped fiber is pumped by feeding the light from a laser pump diode, through a coupler, into the erbium ion doped fiber.

- Optical oscillators are inserted at the entry and exit to allow ONLY optical signals at 1550 nm to pass in one direction and prevent the 980 nm pump light from propagating back or forward into the communication system.
- Energy level E1, E2, and E3 are not single unit levels, but rather consists of closely spaced collection of several levels → range of stimulated transitions from E2 to E1 (1525 – 1565 nm) with 40 nm optical bandwidth → wavelength division multiplexed system (WDM) systems.

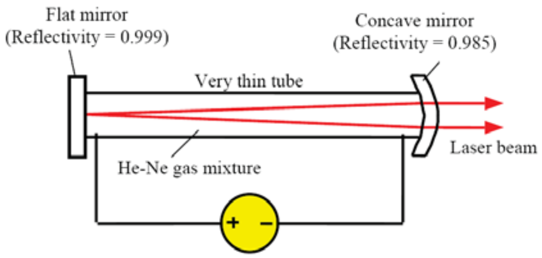
Laser: Basic Principle



A LASER SYSTEM

Laser: Gas Lasers

He-Ne Lasers



Current regulated HV DC/RF power supply
He atom to become excited by collision with drifting electrons

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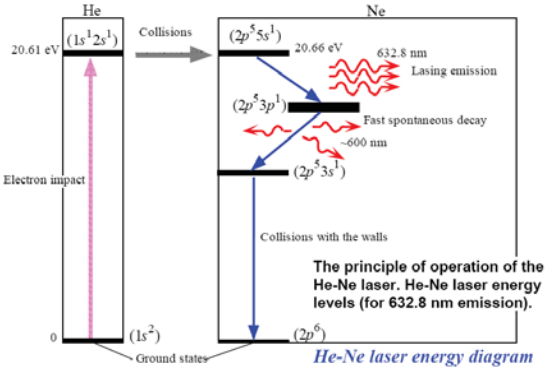
Laser: Gas Lasers

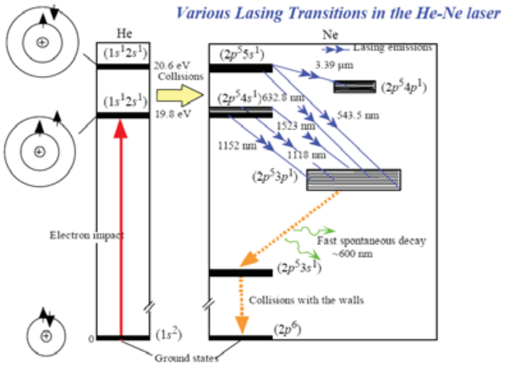
He-Ne Lasers



The principle of operation of the He-Ne laser. He-Ne laser energy levels (for 632.8 nm emission).

He-Ne laser energy diagram

Laser: Gas Lasers



Overall Efficiency

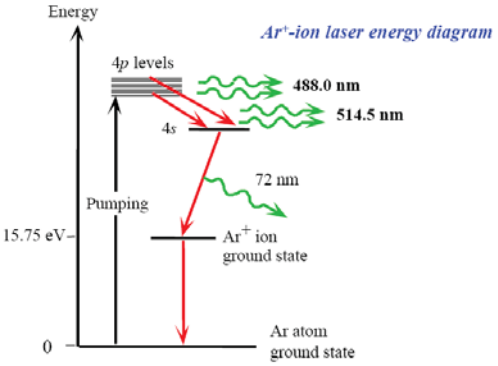
$$\text{Overall efficiency} = \frac{\text{Optical Power Output}}{\text{Electrical Power Input}} \times 100\%$$

Typical commercial He-Ne laser characteristics

Wavelength (nm)	543.5	594.1	612	632.8	1523
	Green	Yellow		Red	Infrared
Optical output power (mW)	1.5	2	4	5	1
Typical current (mA)	6.5	6.5	6.5	6.5	6
Typical voltage	2750	2070	2070	1910	3380
Overall efficiency = P_{out}/IV	0.0084 %	0.015 %	0.030 %	0.040 %	0.005 %

Laser: Gas Lasers

Ar⁺-ion Lasers



Laser: He-Ne Gas Lasers

- By using dc or RF high voltage, electrical discharge is obtained within the tube which causes the He atoms to become excited by collisions with the drifting electrons,

  $He + e^- \rightarrow He^* + e^-$
- The excited He atom, He^*, cannot spontaneously emit a photon → large number of He^* atoms build up during the electrical discharge.
- When He^* collides with a Ne atom, it transfers its energy to the Ne atom by resonance energy exchange.

  $He^* + Ne \rightarrow He + Ne^*$
- A spontaneous emission of a photon from one Ne^* atom gives rise to an avalanche of stimulated emission process → lasing emission with a wavelength 632.8 nm in the red.

Gas Laser Output Spectrum

- Doppler effect → resulting the broadening of the emitted spectrum → output radiation from gas laser covers a spectrum of wavelengths with a central peak.
- Given the average K.E. of (3/2)kT, radiation freq. υ_o (as source frequency), due to Doppler effect, when gas atom is moving away from the observer, the latter detects a lower frequency υ_1 $\upsilon_1 = \upsilon_o\left(1 - \frac{v_x}{c}\right)$

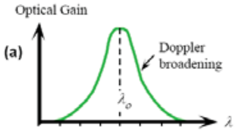
  where v_x is the relative velocity of the atom along the laser tube (x-axis) with respect to observer.
- When atom moving towards the observer, the detected freq υ_2 is higher:

  $$\upsilon_2 = \upsilon_o\left(1 + \frac{v_x}{c}\right)$$

Gas Laser Output Spectrum

- Since the atoms are in random motion the observer will detect a range of frequencies due to Doppler effect.
- Resulting the frequency or wavelength of the output radiation from a gas laser will have a "linewidth" $\Delta\upsilon = \upsilon_2 - \upsilon_1$. It is called **Doppler broadened linewidth**.
- → Stimulated emission wavelength of lasing medium or optical gain has distribution around $\lambda_o = c/\upsilon_o$.
- The full width at half maximum FWHM in the output intensity vs. frequency spectrum is:

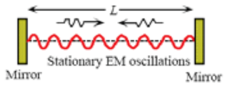
$$\Delta\upsilon_{1/2} = 2\upsilon_o\sqrt{\frac{2kT\ln(2)}{Mc^2}}$$



where M is mass of lasing atom or molecule

Gas Laser Output Spectrum

- Let consider an optical cavity of length L with parallel end mirrors (etalon – Fabry-Perot optical resonator).



- Any standing wave in the cavity must have an integer number of half-wavelengths $\lambda/2$ that fit into the cavity length L,

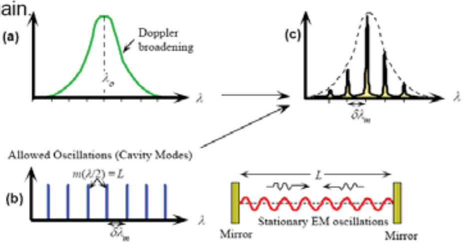
$$m\left(\frac{\lambda}{2}\right) = L$$

where m is mode number of the standing wave.

- **Cavity mode**: each possible standing wave within the cavity (laser tube) which satisfy the above equation.
- **Axial (longitudinal) modes**: existing modes along the cavity axis.

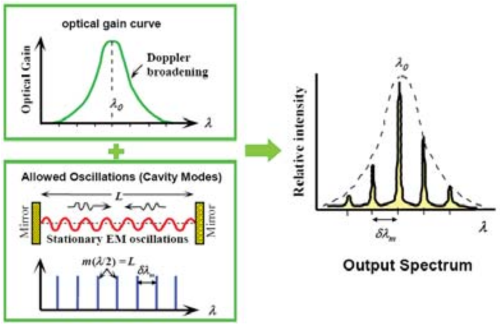
Gas Laser Output Spectrum

- The laser output thus has broad spectrum with peaks at certain wavelengths corresponding to various cavity modes existing within the Doppler broadened optical gain.



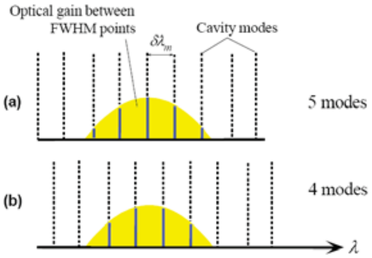
The output spectrum is determined by satisfying (a) and (b) simultaneously.

Gas Laser Output Spectrum



Output Spectrum

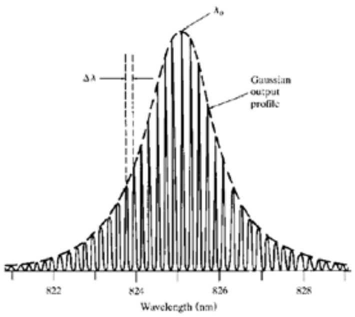
Optical Gain



Number of laser modes depends on how the cavity modes intersect the optical gain curve. In this case we are looking at modes within the linewidth $\Delta\lambda_{1/2}$.

Optical Gain

Fabry-Perot laser spectrum



Example

A typical low power 5mW He-Ne laser tube operate at a DC voltage of 2000V and carrier a current of 7mA . What is the efficiency of the laser?

Solution:

Efficiency=output light power/Input Electric power
$=5 \times 10^{-3}$W/(7×101^{-3}A)(2000V)
=0.036%
Note that 5mW over a beam diameter of 1mm is 6.4kW/m^{-2}

Example

4.2 The He-Ne Laser A particular He-Ne laser operating at 632.8 nm has a tube that is 50 cm long. The operating temperature is 130 °C

a Estimate the Doppler broadened linewidth ($\Delta\lambda$ in the output spectrum.

b What are the mode number m values that satisfy the resonant cavity condition? How many modes are therefore allowed?

c What is the separation $\Delta\nu_m$ in the frequencies of the modes? What is the mode separation $\Delta\lambda_m$ in wavelength.

Solution

a The central emission frequency is

$$\upsilon_o = c/\lambda_o = (3\times10^8 \text{ m s}^{-1}) / (632.8\times10^{-9} \text{ m}) = 4.74\times10^{14} \text{ s}^{-1}.$$

The FWHM width of the frequencies $\Delta\upsilon_{1/2}$ observed will be given by Eq. (3)

$$\Delta\upsilon_{1/2} = 2\upsilon_o\sqrt{\frac{2k_BT\ln(2)}{Mc^2}} = 2(4.748\times10^{14})\sqrt{\frac{2(1.38\times10^{-23})(130+273)\ln(2)}{(3.35\times10^{-26})(3\times10^8)^2}}$$

$$= 1.515 \text{ GHz}$$

To get FWHM wavelength width $\Delta\lambda_{1/2}$, differentiate $\lambda = c/\nu$

$$\frac{d\lambda}{d\upsilon} = \frac{c}{\upsilon^2} = \frac{\lambda}{\upsilon}$$

so that $\Delta\lambda_{1/2} \approx \Delta\upsilon_{1/2}|-\lambda/\upsilon| = (1.515\times10^9 \text{ Hz})(632.8\times10^{-9} \text{ m}) / (4.74\times10^{14} \text{ s}^{-1})$

or $\Delta\lambda_{1/2} \approx 2.02\times10^{-12}$ m or 0.00202 nm.

This width is between the half-points of the spectrum.

Solution

b For $\lambda = \lambda_o = 632.8$ nm, the corresponding mode number m_o is,

$$m_o = 2L / \lambda_o = (2\times0.5 \text{ m}) / (632.8\times10^{-9} \text{ m}) = 1580278.1$$

and actual m_o has to be the closest integer value to 1580278.1, that is 1580278

Consider the minimum and maximum wavelengths corresponding to the extremes of the spectrum at the half-power points:

$$\lambda_{\min} = \lambda_o - \tfrac{1}{2}\Delta\lambda = 632.798987$$

and

$$\lambda_{\max} = \lambda_o + \tfrac{1}{2}\Delta\lambda = 632.801012$$

c The frequency separation $\Delta\upsilon_m$ of two consecutive modes is

$$\Delta\upsilon_m = \upsilon_{m+1} - \upsilon_m = \frac{c}{\lambda_{m+1}} - \frac{c}{\lambda_m} = \frac{c}{\frac{2L}{(m+1)}} - \frac{c}{\frac{2L}{m}} = \frac{c}{2L}$$

or

$$\Delta\upsilon_m = \frac{c}{2L} = \frac{3\times10^8}{2(0.5)} = \mathbf{3\times10^8\ Hz.}$$

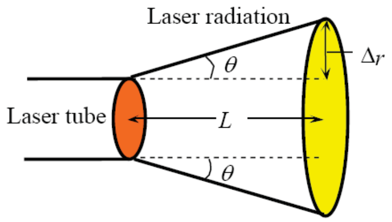
The wavelength separation of two consecutive modes is

$$\Delta\lambda_m = \frac{\lambda_m^2}{2L} = \frac{(632.8\times10^{-9})^2}{2(0.5)} = 4.004\times10^{-13} \text{ m or } \mathbf{0.4004\ pm}.$$

Note:

$$\text{Modes} = \frac{\text{Linewidth of spectrum}}{\text{Separation of two modes}} \approx \frac{\Delta\lambda_{1/2}}{\Delta\lambda_m} = \frac{2.02 \text{ pm}}{0.4004 \text{ pm}} = 5.04.$$

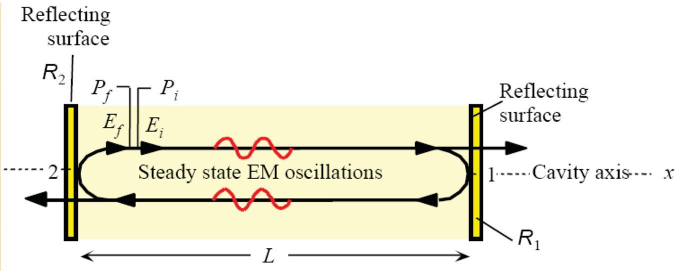
Laser Beam Divergence



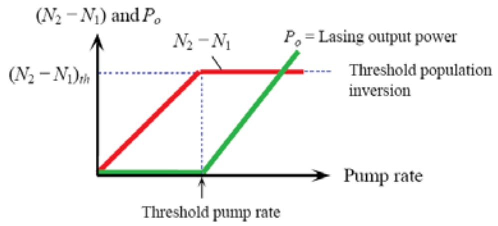
The output laser beam has a divergence characterized by the angle 2θ (highly exaggerated in the figure)

Δr=Ltanθ. What is the diameter of the beam at a distance of 10m, if divergence is 1mrad?

Optical Cavity Resonator



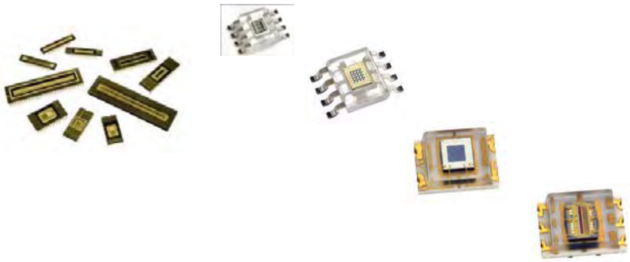
Laser Oscillator



Simplified description of a laser oscillator. $(N_2 - N_1)$ and coherent output power (P_o) vs. pump rate under continuous wave steady state operation.

Photodetectors

Photodiodes



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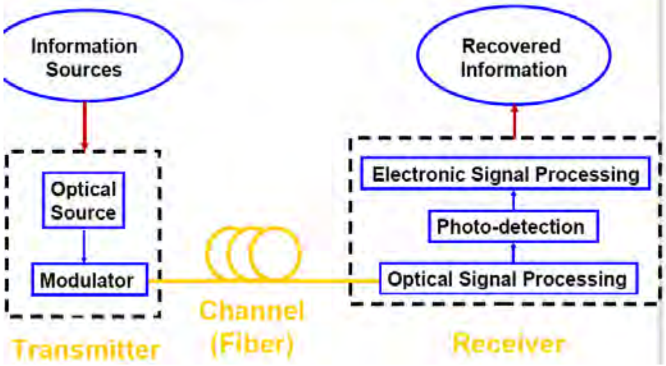
1. Introduction



Information Sources

Optical Source

Modulator

Transmitter

Channel (Fiber)

Recovered Information

Electronic Signal Processing

Photo-detection

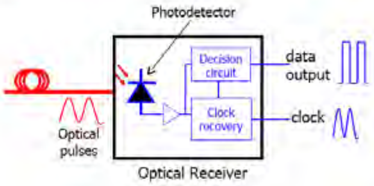
Optical Signal Processing

Receiver

---

4. Photodetector

- ☐ Convert an optical signal into an electrical signal
  - Photodetectors made of semiconductor materials absorb incident photons and produces electrons
  - If electric field imposed on photodector an electric current (photocurrent) is produced ⇒ photodiode



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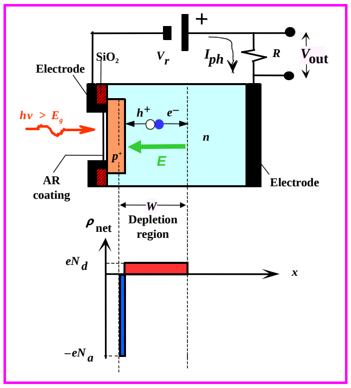
4. Photodetector

- ☐ Basic requirements of a photodetector
  - Sensitivity at the required wavelength
  - Efficient conversion of photons to electrons
  - Fast response to operate at high frequencies
  - Low noise for reduced errors
  - Sufficient area for efficient coupling to optical fiber
  - High reliability
  - Low cost

Photo-detectors: Principle of the P-N junction photo-diode

- ☐ Schematic diagram of a reverse biased p-n junction photodiode
  - ➢ Photocurrent is depend on number of EHP and drift velocity.
  - ➢ The electrode do not inject carriers but allow excess carriers in the sample to leave and become collected by the battery.
- ☐ Net space charge across the diode in the depletion region. N_d and N_a are the donor and acceptor concentrations in the *p* and *n* sides.



Principle of *pn* junction photodiode

(a) Reversed biased *pn* junction photodiode.

- Annular electrode to allow photon to enter the device.
- Anti-reflection coating (Si_3N_4) to reduce the reflection.
- The p^+-side thickness < 1 μm.

(b) Net space charge distribution, within SCL.

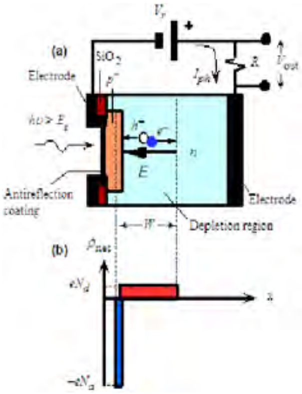
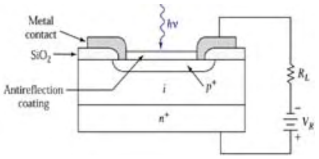
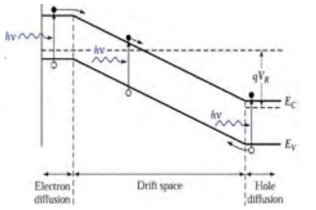


Photo-detectors: Principle of the p-n junction Photo-diode

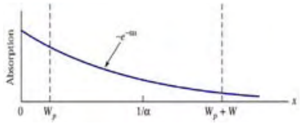
- ☐ Operation of a photo-diode



(a) Cross-section view of a photo-diode



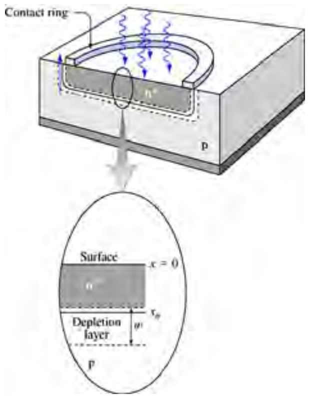
(b) Energy band diagram under reverse bias.



(c) Carrier absorption characteristics.

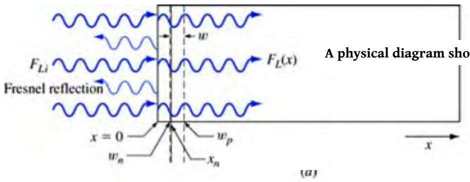
Photodetectors: Principle of the p-n junction Photo-diode

- ☐ A generic photo-diode

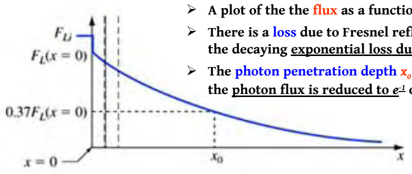


Photodetectors: Principle of the p-n junction Photo-diode

- ☐ Variation of photon flux with distance.



A physical diagram showing the depletion region.



- ➢ A plot of the the flux as a function of distance.
- ➢ There is a loss due to Fresnel reflection at the surface, followed by the decaying exponential loss due to absorption.
- ➢ The photon penetration depth x_0 is defined as the depth at which the photon flux is reduced to e^{-1} of its surface value.

Photo-detectors: RAMO's Theorem and External Photo-current

- ➢ An EHP is photogenerated at $x = l$. The electron and the hole drift in opposite directions with drift velocities v_h and v_e.
- ➢ The electron arrives at time $t_{electron} = (L-l)/v_e$ and the hole arrives at time $t_{hole} = l/v_h$.

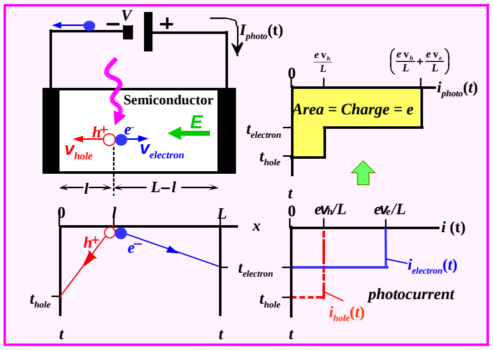


Photo-detectors: RAMO's Theorem and External Photo-current

- ➢ As the electron and hole drift, each generates $i_{electron}(t)$ and $i_{hole}(t)$.
- ➢ The total photocurrent is the sum of hole and electron photocurrents each lasting a duration t_h and t_e respectively.

$$t_e(t) = \frac{L-l}{v_e} \quad and \quad t_h(t) = \frac{l}{v_h} \qquad \textit{Transit time}$$

$$\text{Work done} = e \cdot E\, dx = V \cdot i_e(t)\, dt \qquad \boxed{E} = \frac{V}{L} \qquad v_e = \boxed{\frac{dx}{dt}}$$

$$\boxed{i_e(t) = \frac{e\,v_e}{L}; \quad t < t_e \qquad i_h(t) = \frac{e\,v_h}{L}; \quad t < t_h \qquad \textit{Photocurrent}}$$

$$Q_{collected} = \int_0^{t_e} i_e(t)\, dt + \int_0^{t_h} i_h(t)\, dt = \boxed{e}$$

The collected charge is not 2e but just "one electron".

If a charge q is being drifted with a velocity $v_d(t)$ by a field between two biased electrodes separated by L, the motion of q generates an <u>external current</u> given by

$$\boxed{i(t) = \frac{e\,v_d(t)}{L}; \quad t < t_{transit} \qquad \textit{Ramo's Theorem}}$$

Photo-detectors: Absorption Coefficient & Photo-diode Materials

- ☐ Absorbed Photon create Electron-Hole Pair.

$$\boxed{\lambda_g[\mu m] = \frac{1.24}{E_g[eV]} \qquad \textit{Cut-off wavelength vs. Energy bandgap}}$$

- ☐ Incident photons become absorbed as they travel in the semiconductor and <u>light intensity decays exponentially</u> with distance into the semiconductor.

$$\boxed{I(x) = I_0 \cdot e^{\boxed{-\alpha} x} \qquad \textit{Absorption coefficient}}$$

Absorption Coefficient

- Absorption coefficient α is a material property.
- Most of the photon absorption (63%) occurs over a distance 1/α (it is called **penetration depth** δ)

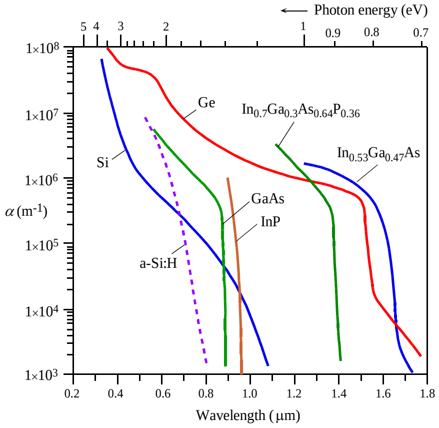
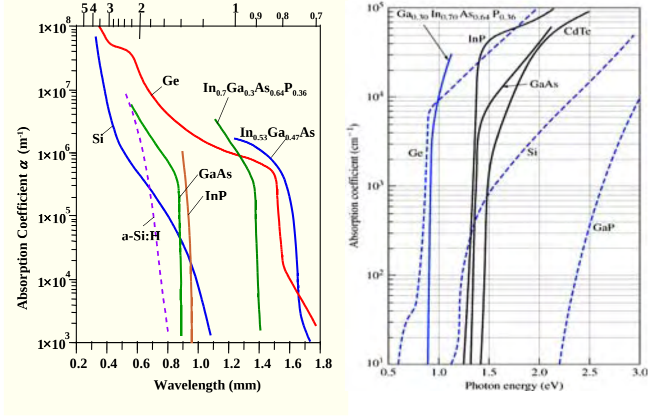


Photo-detectors: Absorption Coefficient & Photo-diode Materials

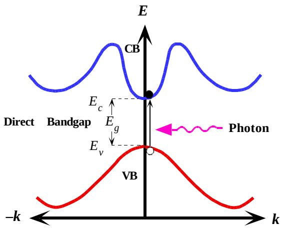
❑ Absorption

The indirect-gap materials are shown with a broken line.



Absorption Coefficient

- Direct bandgap semiconductors (GaAs, InAs, InP, GaSb, InGaAs, GaAsSb), the photon absorption does not require assistant from lattice vibrations. The photon is absorbed and the electron is excited directly from the VB to CB without a change in its k-vector (crystal momentum ħk), since photon momentum is very small.



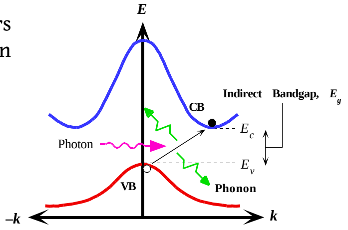
(a) GaAs (Direct bandgap)

$$\hbar k_{CB} - \hbar k_{VB} = \text{photon momentum} \approx 0$$

- Absorption coefficient α for direct band-gap semiconductors rise sharply with decreasing wavelength from λ_g (GaAs and InP).

Absorption Coefficient

- Indirect band-gap semiconductors (Si and Ge), the photon absorption requires assistant from lattice vibrations (**phonon**). If K is wave vector of lattice wave, then ħK represents the momentum associated with lattice vibration → ħK is a **phonon momentum.**



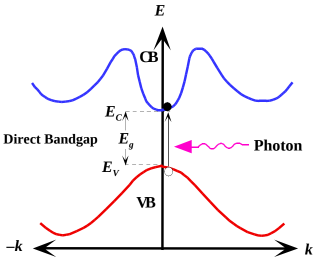
(b) Si (Indirect bandgap)

$$\hbar k_{CB} - \hbar k_{VB} = \text{phonon momentum} = \hbar K$$

- Thus the probability of photon absorption is not as high as in a direct transition and the λ_g is not as sharp as for direct band-gap semiconductors.

Photo-detectors: Absorption Coefficient & Photo-diode Materials

Photon absorption in a direct bandgap semiconductor.



Photon absorption in an indirect bandgap semiconductor

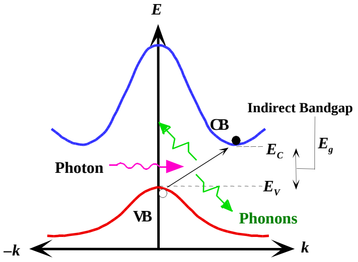


Photo-detectors: Quantum Efficiency and Responsivity

❑ External Quantum Efficiency

$$\eta = \frac{\text{Number of EHP generated and collected}}{\text{Number of incidnet photons}} = \frac{I_{ph}/e}{P_0/h\nu}$$

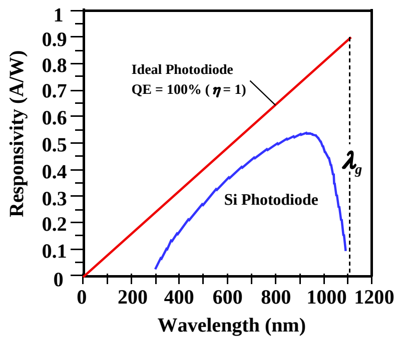
❑ Responsivity

$$R = \frac{\text{Photocurrent (A)}}{\text{Incident Optical Power (W)}} = \frac{I_{ph}}{P_0}$$

$$R = \eta \frac{e}{h\nu} = \eta \frac{e\lambda}{hc} \quad \textit{Spectral Responsivity}$$

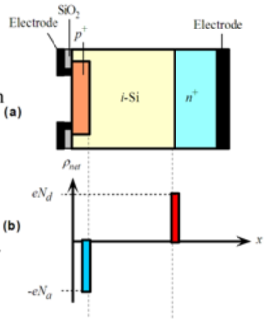
Photo-detectors

- ❑ **Responsivity vs. wavelength for a typical *Si photo-diode***



The *pin* Photo-diode

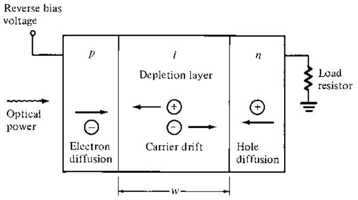
- The *pn* junction photo-diode has two drawbacks:
  - Depletion layer capacitance is not sufficiently small to allow photodetection at high modulation frequencies (RC time constant limitation).
  - Narrow SCL (at most a few microns) → long wavelengths incident photons are absorbed outside SCL → low QE
- The *pin* photo-diode can significantly reduce these problems.



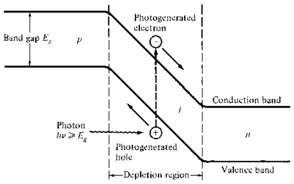
- Intrinsic layer has less doping and wider region (5 – 50 μm).

Photo-detectors: *PIN* Photo-diode

- ❑ **Reverse-biased *p-i-n* photodiode**



- ❑ ***pin* energy-band diagram**



- ❑ ***pin* photodiode circuit**

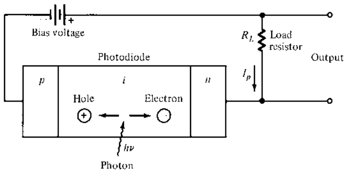
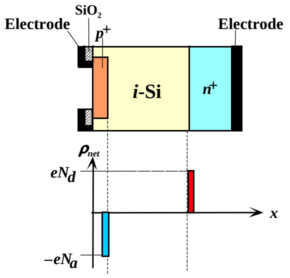
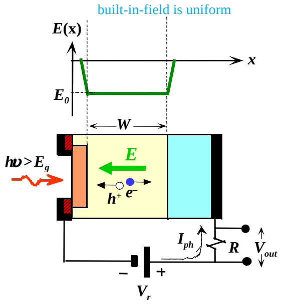


Photo-detectors: *PIN* Photo-diode

- ❑ **Schematic diagram of *pin* photodiode**





- ➢ **Small depletion layer capacitance gives high modulation frequencies.**
- ➢ **High Quantum efficiency.**

Photo-detectors: *PIN* Photo-diode

- ➢ **A reverse biased *pin* photodiode is illuminated with a short wavelength photon that is absorbed very near the surface.**
- ➢ **The photogenerated electron has to diffuse to the depletion region where it is swept into the *i*- layer and drifted across.**

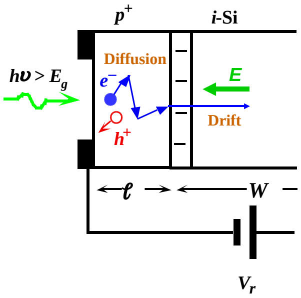
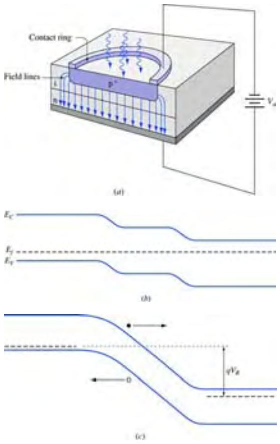


Photo-detectors: *PIN* Photo-diode



***p-i-n* diode**

(a) The structure;

(b) equilibrium energy band diagram;

(c) energy band diagram under reverse bias.

Photo-detectors: *PIN* Photo-diode

- ❑ **The responsivity of *PIN* photodiodes**

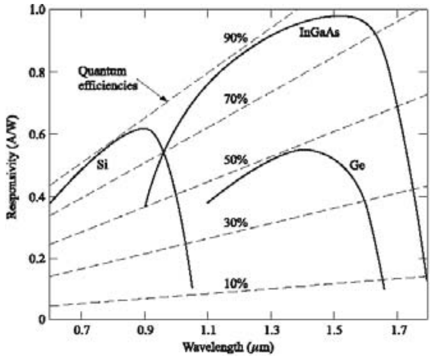


Photo-detectors: Photo-conductive Detectors and Gain

- ❑ **Quantum efficiency versus wavelength for various photo-detectors**

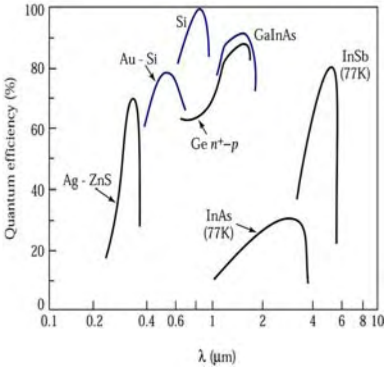


Photo-detectors: *PIN* Photo-diode

- ❑ **<u>Junction capacitance</u> of *pin***

$$\boxed{C_{dep} = \frac{\varepsilon_0 \varepsilon_r A}{W}}$$

- ➢ **Small capacitance: High modulation frequency**
- ➢ **RC_{dep} time constant is ~ 50 psec.**

- ❑ **<u>Electric field</u> of biased *pin***

$$\boxed{E = E_0 + \frac{V_r}{W} \approx \frac{V_r}{W}}$$

- ❑ **<u>Response time</u>**

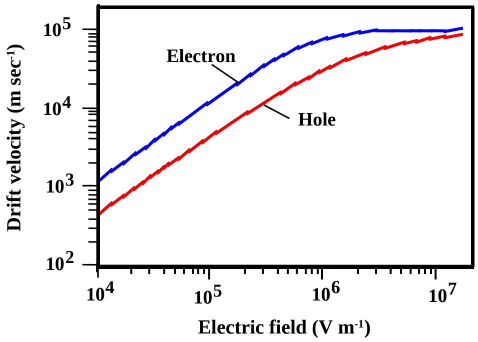
$$\boxed{\begin{aligned} t_{drift} &= \frac{W}{\mathrm{v}_d} \\ \mathrm{v}_d &= \mu_d E \end{aligned}}$$

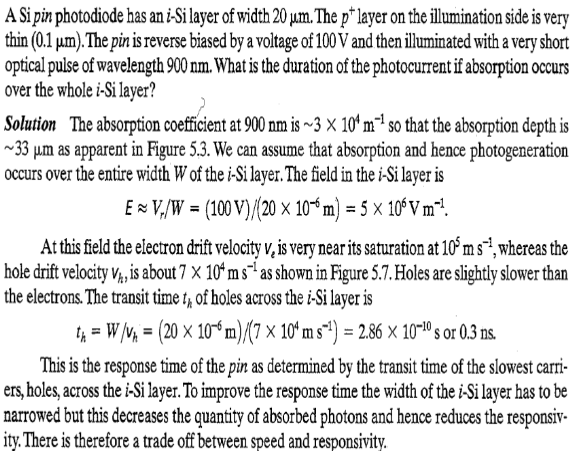
The <u>speed of pin photodiodes</u> are invariably limited by the <u>transit time of photogenerated carriers across the *i*-Si layer</u>.

For *i*-Si layer of width 10 μm, the drift time is about 0.1 nsec.

Photo-detectors: *PIN* Photo-diode

- ❑ **Drift velocity vs. electric field for holes and electrons in Silicon.**





A Si *pin* photodiode has an *i*-Si layer of width 20 μm. The p^+ layer on the illumination side is very thin (0.1 μm). The *pin* is reverse biased by a voltage of 100 V and then illuminated with a very short optical pulse of wavelength 900 nm. What is the duration of the photocurrent if absorption occurs over the whole *i*-Si layer?

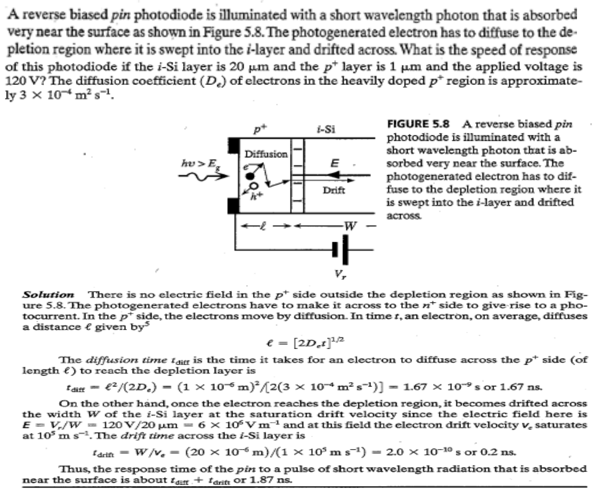
Solution The absorption coefficient at 900 nm is ~3×10^4 m^{-1} so that the absorption depth is ~33 μm as apparent in Figure 5.3. We can assume that absorption and hence photogeneration occurs over the entire width *W* of the *i*-Si layer. The field in the *i*-Si layer is

$$E \approx V_r/W = (100\,\mathrm{V})/(20 \times 10^{-6}\,\mathrm{m}) = 5 \times 10^6\,\mathrm{V\,m^{-1}}.$$

At this field the electron drift velocity v_e is very near its saturation at 10^5 m s^{-1}, whereas the hole drift velocity v_h, is about 7×10^4 m s^{-1} as shown in Figure 5.7. Holes are slightly slower than the electrons. The transit time t_h of holes across the *i*-Si layer is

$$t_h = W/v_h = (20 \times 10^{-6}\,\mathrm{m})/(7 \times 10^4\,\mathrm{m\,s^{-1}}) = 2.86 \times 10^{-10}\,\mathrm{s}\ \text{or}\ 0.3\,\mathrm{ns}.$$

This is the response time of the *pin* as determined by the transit time of the slowest carriers, holes, across the *i*-Si layer. To improve the response time the width of the *i*-Si layer has to be narrowed but this decreases the quantity of absorbed photons and hence reduces the responsivity. There is therefore a trade off between speed and responsivity.



A reverse biased *pin* photodiode is illuminated with a short wavelength photon that is absorbed very near the surface as shown in Figure 5.8. The photogenerated electron has to diffuse to the depletion region where it is swept into the *i*-layer and drifted across. What is the speed of response of this photodiode if the *i*-Si layer is 20 μm and the p^+ layer is 1 μm and the applied voltage is 120 V? The diffusion coefficient (D_e) of electrons in the heavily doped p^+ region is approximately 3×10^{-4} m^2 s^{-1}.

FIGURE 5.8 A reverse biased *pin* photodiode is illuminated with a short wavelength photon that is absorbed very near the surface. The photogenerated electron has to diffuse to the depletion region where it is swept into the *i*-layer and drifted across.

Solution There is no electric field in the p^+ side outside the depletion region as shown in Figure 5.8. The photogenerated electrons have to make it across to the n^+ side to give rise to a photocurrent. In the p^+ side, the electrons move by diffusion. In time *t*, an electron, on average, diffuses a distance ℓ given by[5]

$$\ell = [2D_e t]^{1/2}$$

The *diffusion time* t_{diff} is the time it takes for an electron to diffuse across the p^+ side (of length ℓ) to reach the depletion layer is

$$t_{\mathrm{diff}} = \ell^2/(2D_e) = (1 \times 10^{-6}\,\mathrm{m})^2/[2(3 \times 10^{-4}\,\mathrm{m^2\,s^{-1}})] = 1.67 \times 10^{-9}\,\mathrm{s}\ \text{or}\ 1.67\,\mathrm{ns}.$$

On the other hand, once the electron reaches the depletion region, it becomes drifted across the width *W* of the *i*-Si layer at the saturation drift velocity since the electric field here is $E = V_r/W = 120\,\mathrm{V}/20\,\mu\mathrm{m} = 6 \times 10^6\,\mathrm{V\,m^{-1}}$ and at this field the electron drift velocity v_e saturates at 10^5 m s^{-1}. The *drift time* across the *i*-Si layer is

$$t_{\mathrm{drift}} = W/v_e = (20 \times 10^{-6}\,\mathrm{m})/(1 \times 10^5\,\mathrm{m\,s^{-1}}) = 2.0 \times 10^{-10}\,\mathrm{s}\ \text{or}\ 0.2\,\mathrm{ns}.$$

Thus, the response time of the *pin* to a pulse of short wavelength radiation that is absorbed near the surface is about $t_{\mathrm{diff}} + t_{\mathrm{drift}}$ or 1.87 ns.

A Si *pin* photodiode has an active light receiving area of diameter 0.4 mm. When radiation of wavelength 700 nm (red light) and intensity 0.1 mW cm^{-2} is incident it generates a photocurrent of 56.6 nA. What is the responsivity and QE of the photodiode at 700 nm?

Solution The incident light intensity $I = 0.1$ mW cm^{-2} means that the incident power for conversion is

$$P_o = AI = \pi(0.02\ \text{cm})^2(1 \times 10^{-3}\ \text{W cm}^{-2}) = 1.26 \times 10^{-7}\ \text{W or } 0.126\ \mu\text{W}.$$

The responsivity is

$$R = I_{ph}/P_o = (56.6 \times 10^{-9}\ \text{A})/(1.26 \times 10^{-7}\ \text{W}) = 0.45\ \text{A W}^{-1}$$

The QE can be found from

$$\eta = R\frac{hc}{e\lambda} = (0.45\ \text{A W}^{-1})\frac{(6.62 \times 10^{-34}\ \text{J s})(3 \times 10^8\ \text{m s}^{-1})}{(1.6 \times 10^{-19}\ \text{C})(700 \times 10^{-9}\ \text{m})} = 0.80 = 80\%$$

Example

Bandgap and photodetection

(a) Determine the maximum value of the energy gap which a semiconductor, used as a photoconductor, can have if it is to be sensitive to yellow light (600 nm).

(b) A photodetector whose area is 5×10^{-2} cm^2 is irradiated with yellow light whose intensity is 20 mW cm^{-2}. Assuming that each photon generates one electron-hole pair, calculate the number of pairs generated per second.

Solution

(a) Given, λ = 600 nm, we need $E_{ph} = h\upsilon = E_g$ so that,

$E_g = hc/\lambda$ = (6.626×10^{-34} J s)(3×10^8 m s^{-1})/(600×10^{-9} m) = 2.07 eV

(b) *Area* = 5×10^{-2} cm^2 and I_{light} = 20×10^{-3} W/cm^2.

The received power is

P = Area ×I$_{light}$ = (5×10^{-2} cm^2)(20×10^{-3} W/cm^2) = 10^{-3} W

N_{ph} = number of photons arriving per second = P/E_{ph}

= (10^{-3} W)/(2.059×1.60218×10^{-19} J/eV)

= 2.9787×10^{15} photons s^{-1} = 2.9787×10^{15} EHP s^{-1}.

Example

Bandgap and Photodetection

(c) From the known energy gap of the semiconductor GaAs (E_g = 1.42 eV), calculate the primary wavelength of photons emitted from this crystal as a result of electron-hole recombination. Is this wavelength in the visible?

(d) Will a silicon photodetector be sensitive to the radiation from a GaAs laser? Why?

Solution

(c) For GaAs, E_g = 1.42 eV and the corresponding wavelength is $\lambda = hc/\ E_g$ = (6.626×10^{-34} J s)(3×10^8 m s^{-1})/(1.42 eV×1.6×10^{-19} J/eV) = 873 nm (invisible IR) ∴ The wavelength of emitted radiation due to EHP recombination is **873 nm.**

(d) For Si, E_g = 1.1 eV and the corresponding cut-off wavelength is $\lambda_g = hc/\ E_g$ = (6.626×10^{-34} J s)(3×10^8 m s^{-1})/(1.1 eV×1.6×10^{-19} J/eV) = **1120 nm**
Since the 873 nm wavelength is shorter than the cut-off wavelength of 1120 nm, the Si photodetector can detect the 873 nm radiation (Put differently, the photon energy corresponding to 873 nm, 1.42 eV, is larger than the E_g, 1.1 eV, of Si which mean that the Si photodetector can indeed detect the 873 nm radiation)

Example

Absorption coefficient

(a) If *d* is the thickness of a photodetector material, I_o is the intensity of the incoming radiation, the number of photons absorbed per unit volume of sample is

$$n_{ph} = \frac{I_0[1-\exp(-\alpha\cdot d)]}{d\,h\upsilon}$$

Solution

(a) If I_o is the intensity of incoming radiation (energy flowing per unit area per second), $I_o\exp(-\alpha d)$ is the transmitted intensity through the specimen with thickness d and thus $I_o\exp(-\alpha d)$ is the "absorbed" intensity

Example

(b) What is the thickness of a Ge and In$_{0.53}$Ga$_{0.47}$As crystal layer that is needed for absorbing 90% of the incident radiation at 1.5 µm?

For Ge, $\alpha \approx 5.2 \times 10^5\ m^{-1}$ at 1.5 µm incident radiation.
For In$_{0.53}$Ga$_{0.47}$As, $\alpha \approx 7.5 \times 10^5\ m^{-1}$ at 1.5 µm incident radiation.

(b) **For Ge, $\alpha \approx 5.2 \times 10^5\ m^{-1}$ at 1.5 µm incident radiation.**

$$\therefore\quad 1-\exp(-\alpha\cdot d) = 0.9$$

$$d = \frac{1}{\alpha}\ln\left(\frac{1}{1-0.9}\right) = \frac{1}{5.2\times10^5}\ln\left(\frac{1}{1-0.9}\right) = 4.428\times10^{-6}\,m = 4.428\,\mu m$$

For In$_{0.53}$Ga$_{0.47}$As, $\alpha \approx 7.5 \times 10^5\ m^{-1}$ at 1.5 µm incident radiation.

$$d = \frac{1}{7.5\times10^5}\ln\left(\frac{1}{1-0.9}\right) = 3.07\times10^{-6}\,m = 3.07\,\mu m$$

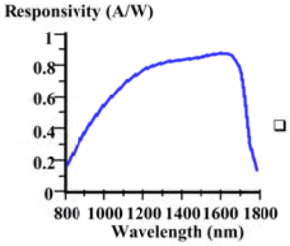
Example

InGaAs *pin* Photodiodes

Consider a commercial InGaAs *pin* photodiode whose responsivity is shown below. Its dark current is 5 nA.

(a) What optical power at a wavelength of 1.55 µm would give a photocurrent that is twice the dark current? What is the QE of the photodetector at 1.55 µm?

(b) What would be the photocurrent if the incident power in **(a)** was at 1.3 µm? What is the QE at 1.3 µm operation?



- The responsivity of an InGaAs *pin* photodiode

Solution

(a) At $\lambda = 1.55\times10^{-6}$ m, from the responsivity vs. wavelength curve we have $R \approx 0.87$ A/W. From the definition of responsivity,

$$R = \frac{Photocurrent\ (A)}{Incident\ Optical\ Power\ (W)} = \frac{I_{ph}}{P_0}$$

we have $$P_0 = \frac{I_{ph}}{R} = \frac{2I_{dark}}{R} = \frac{2\times5\times10^{-9}(A)}{0.87\ A/W} = 11.5\ nW$$

From the definitions of quantum efficiency η and responsivity,

$$R = \eta\frac{e}{h\upsilon} = \eta\frac{e\lambda}{hc}$$

$$\eta = \frac{hcR}{e\lambda} = \frac{(6.62\times10^{-34}\,J\cdot\text{sec})(3\times10^{8}\,m/s)(0.87\,A/W)}{(1.6\times10^{-19}\,coul)(1.55\times10^{-6}\,m)} = 0.70\ (70\ \%)$$

Note the following dimensional identities: A = C s^{-1} and W = J s^{-1} so that A W^{-1} = C J^{-1}. Thus, responsivity in terms of photocurrent per unit incident optical power is also charge collected per unit incident energy.

Solution

(b) At $\lambda = 1.3\times10^{-6}$ m, from the responsivity *vs.* wavelength curve, $R = 0.82$ A/W.

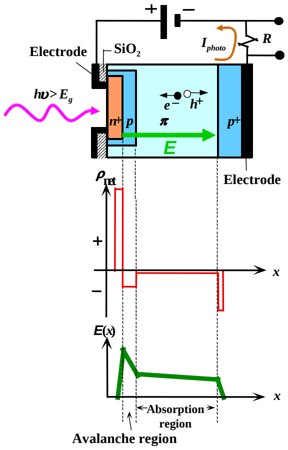
Since P_o is the same and 11.5 nW as in **(a)**,

$$I_{ph} = R\cdot P_0 = (0.82\ A/W)(1.15\ nW) = 9.43\ nA$$

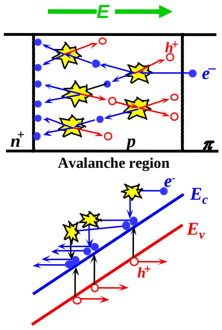
The QE at $\lambda = 1.3$ µm is

$$\eta = \frac{hcR}{e\lambda} = \frac{(6.62\times10^{-34}\,J\cdot\text{sec})(3\times10^{8}\,m/s)(0.82\,A/W)}{(1.6\times10^{-19}\,coul)(1.3\times10^{-6}\,m)} \approx 0.78\ (78\ \%)$$

Photo-detectors: Avalanche Photo-diode (APD)



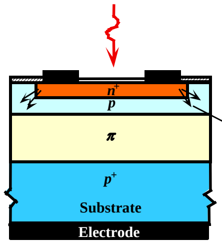
❑ **Impact ionization processes resulting avalanche multiplication**



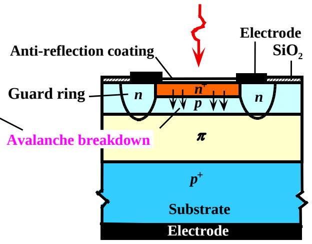
- **Impact of an energetic electron's kinetic energy excites VB electron to the CV.**

Photo detectors: Avalanche Photo-diode (APD)

❑ **Schematic diagram of typical *Si* APD.**



***Si* APD structure without a guard ring**



More practical *Si* APD

- **Breakdown voltage around periphery is higher and avalanche is confined more to illuminated region (n^+p junction).**

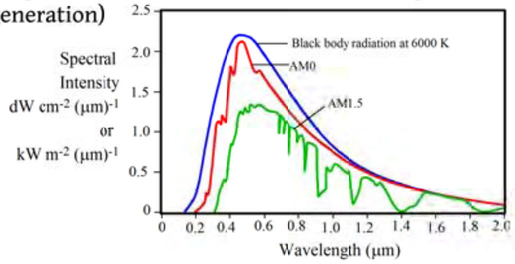
Photodetectors

Solar Cell

- Solar energy spectrum
- Photovoltaic device principles
- I-V Characteristics

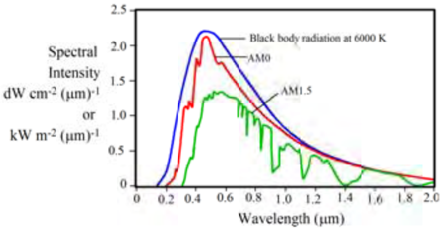
Solar Energy Spectrum

- Photovoltaic devices (solar cells) convert the incident solar radiation energy into electrical energy.
- Absorbed photons → photogeneration → current (photo-current) in external circuit
- Power range: from < mW (calculator) to few MW (photovoltaic power generation)



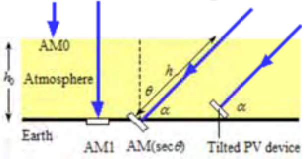
Solar Energy Spectrum

- Spectral intensity I_λ: Intensity per wavelength.
- $I_\lambda \delta\lambda$: intensity in a small interval $\delta\lambda$.
- Total intensity I: integration of I_λ over the whole spectrum.
- **Solar constant** or **air-mass zero** (AM0): the total intensity above earth's atmosphere, approx. constant at 1.353 kW m^{-2}.



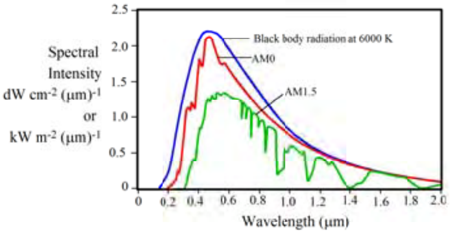
Solar Energy Spectrum

- On sunny day, light intensity on earth's surface is about 70% of the intensity above the atmosphere.
- Absorption and scattering effects increase with the sun beam's path through the atmosphere.
- The shortest path through the atmosphere is when the sun is directly above that location and the received spectrum is called **air mass one** (AM1).
- **Air mass** ***m*** (AM*m*): the ratio of the actual radiation path h to the shortest path h_o, $m = h/h_o$. Since $h = h_o\sec\theta$, AM*m* is AMsecθ.



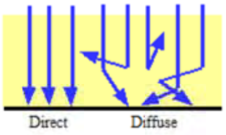
Solar Energy Spectrum

- The spectral distribution AM1.5 has several sharp absorption peaks at certain wavelengths which are due to those wavelength being absorbed by various molecule in the atmosphere, such as ozone, air, and water vapor molecules.
- Dust particles scatter the sun light → reduces the intensity and gives rise to the sun's rays arriving at random angle.

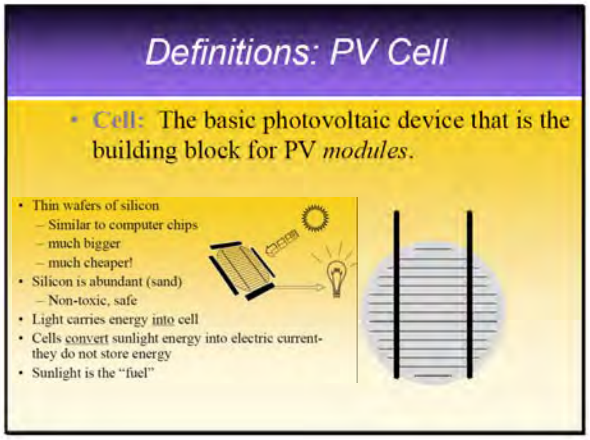


Solar Energy Spectrum

- Thus, the terrestrial light has a **diffuse** component in addition to **direct** component.
- Cloud and sun position → increase diffuse component → spectrum shifted toward the blue light.



- Scattering also increase with decreasing wavelength.
- On a clear day, diffusion component can be about 20% of the total radiation.
- The amount of incident radiation depends on the position of the sun. Flat photovoltaic device flat will receive less solar energy by factor cosθ. However it can be tilted to directly face the sun to maximize the collection efficiency.

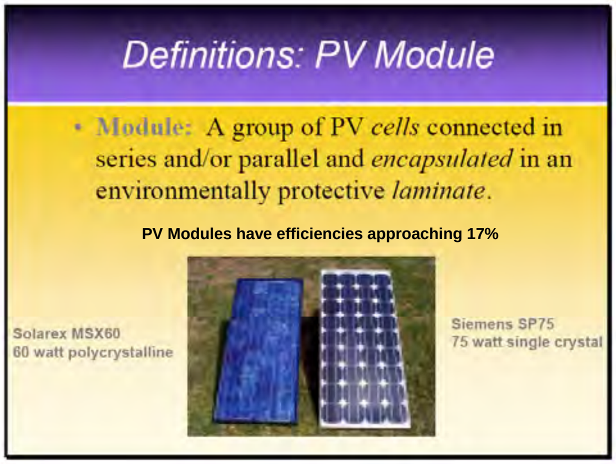


Definitions: PV Cell

- **Cell:** The basic photovoltaic device that is the building block for PV *modules*.

- Thin wafers of silicon
  - Similar to computer chips
  - much bigger
  - much cheaper!
- Silicon is abundant (sand)
  - Non-toxic, safe
- Light carries energy into cell
- Cells convert sunlight energy into electric current- they do not store energy
- Sunlight is the "fuel"

PV Cells have efficiencies approaching 21.5%



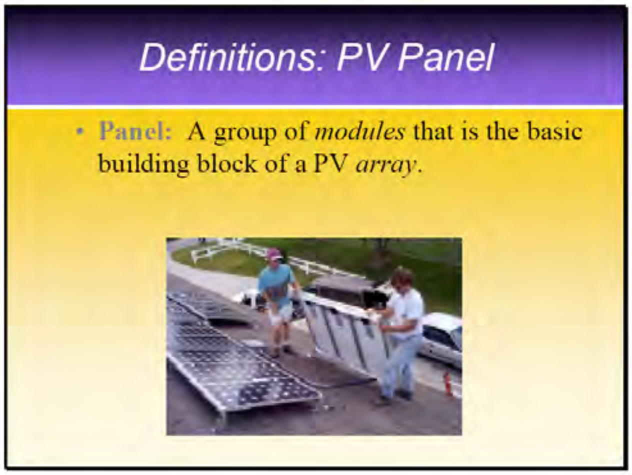
Definitions: PV Module

- **Module:** A group of PV *cells* connected in series and/or parallel and *encapsulated* in an environmentally protective *laminate*.

PV Modules have efficiencies approaching 17%

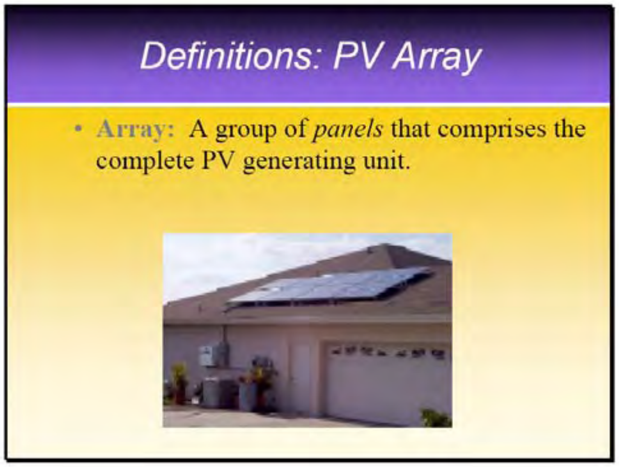
Solarex MSX60
60 watt polycrystalline

Siemens SP75
75 watt single crystal



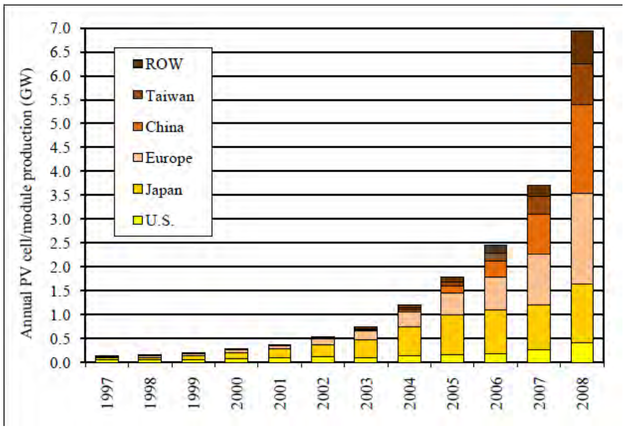
Definitions: PV Panel

- **Panel:** A group of *modules* that is the basic building block of a PV *array*.



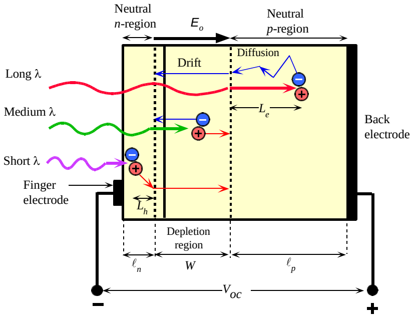
Definitions: PV Array

- **Array:** A group of *panels* that comprises the complete PV generating unit.



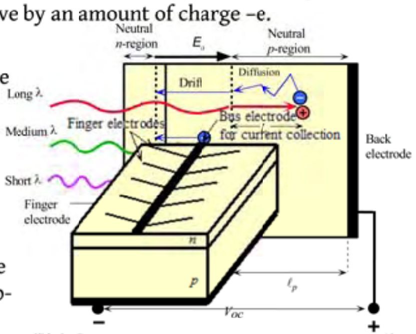
Photovoltaic Device Principles

- Consider pn$^+$ junction with very narrow n-region.
- The illumination is through then thin n-side.
- The SCL extend mainly in p-region with built-in field E_0.
- Electrode at n-side must allow illumination to enter the device and at the same time result in a small series resistance.
- This electrode is formed from array of finger electrodes.



Photovoltaic Device Principles

- Photons are absorbed in SCL within the neutral p-side (l_p) → photogenerated EHP in this region.
- The electron drifts and reaches the neutral n^+ side whereupon it makes this region negative by an amount of charge –e.
- Similarly, hole drifts and reaches the neutral p-side and thereby makes this side positive -> **open circuit voltage** between terminals of the device.
- If there is external load, electron will travel through it and recombine with the excess holes in p-side.

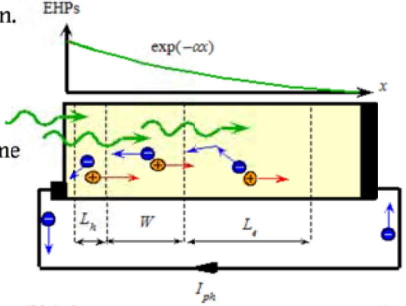


Photovoltaic Device Principles

- Therefore the existence of built-in field E_0 is important to create accumulated electrons in the n-side and holes in the p-side.
- For long wavelength photons → absorbed in the neutral p-side → no E field → diffusion.
- Minority carrier diffusion length L_e.

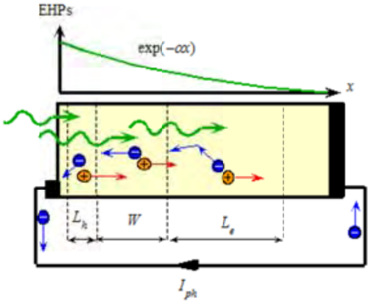
$$L_e = \sqrt{2D_e\tau_e}$$

- τ_e: recombination lifetime of electron.
- D_e: diffusion coefficient in the p-side.



Role of Diffusion Length

- Only those EHPs photogenerated within the L_e to the depletion layer can contribute to the photovoltaic effect.
- Those photogenerated EHPs further away from SCL than L_e are lost by recombination.
- Thus, it is important to have the minority carrier diffusion length L_e as long as possible. By choosing Si pn junction to be p-type which makes electrons to be minority carriers; the electron diffuse length in Si > the hole diffusion length.

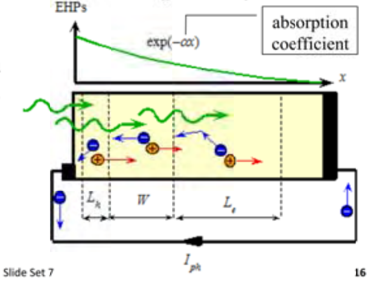


Role of hole diffusion length and s/c current

- For EHPs photogenerated by short-wavelength photons absorbed in the n-side, within diffusion length L_h, can reach SCL and swept across to the p-side.

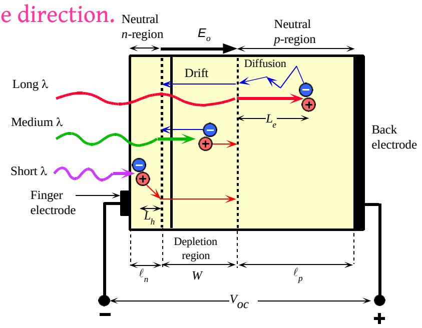
The photogenerated of EHPs that contribute to the photovoltaic effect occurs in a volume of $L_h + W + L_e$.

- If the terminals are shorted then the excess electrons in the n-side can flow through the external circuit to neutralize the excess holes in the p-side -> this current is called **photocurrent**.



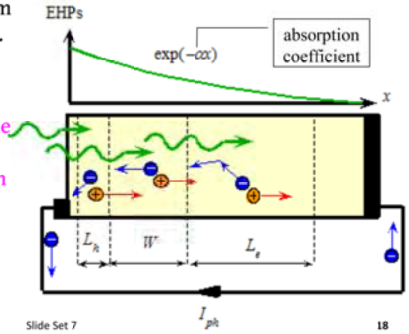
Photovoltaic Device Principles

- Under steady state operation → no net current through an open circuit solar cell → Photocurrent inside the device due to photo generated carriers must be balanced by a flow of carriers in the opposite direction.
- Those are minority carriers that become injected by the appearance of the photovoltaic voltage across the pn junction as in normal diode.



Device optimization and carrier losses

- For long wavelengths, 1 – 1.2 μm, α is small → absorption depth 1/α is typically greater than 100 μm. → Need a thick p-side and long minority carrier diffusion length L_e.
- Thus, p-side is 200 – 500 μm and L_e is shorter than that.
- Si has E_g = 1.1 eV -> correspond to a threshold wavelength of 1.1 μm → The incident energy with wavelength > 1.1μm is then wasted (~ 25%).
- Photons are absorbed and recombined near the crystal surface -> losses.

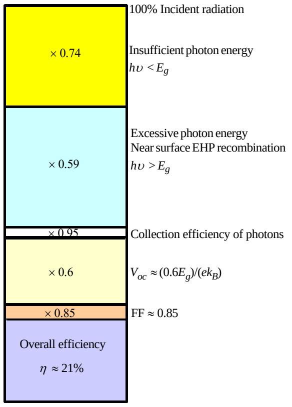


Sources of carrier losses

- Photons are absorbed and recombined near the crystal surface → losses → severely reduce efficiency.
- Crystal surface and interface contain high concentration of **recombination-center**.
- Those facilitate the recombination of photogenerated EHP near the surface.
- The losses due to this event as high as ~ 40%.
- These combined effect bring the efficiency down to about 45%.
- Anti-reflection coating is also contributing the reduction of photons collection due to imperfection with factor of 0.8 – 0.9.
- And including the limitation of photovoltaic action the upper limit to a photovoltaic device that uses single crystal of Si is about 24 – 26% at room temperature.

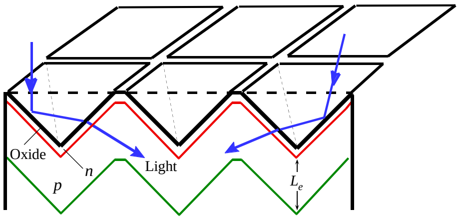
Solar Cells Materials, Devices & Efficiencies

- For a given solar spectrum, conversion efficiency depends on the semiconductor material properties and the device structure.
- Si based solar cell efficiencies 18% for polycrystalline and 22 – 24% for single crystal devices.
- About 25% solar energy is wasted → not enough energy → unable to generate EHPs.
- Considering all losses, the maximum electrical output power is ~20% for a high efficiency Si solar cell.



Solar Cells Materials, Devices & Efficiencies

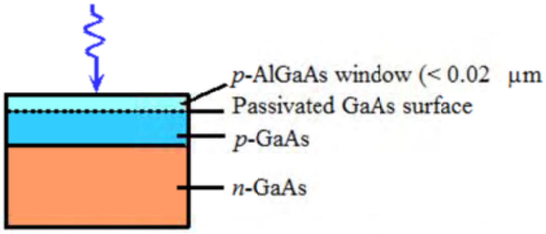
- Si homojunction solar cell efficiencies ~24%. Single crystal PERL (Passivated Emitter Rear Locally-diffused) cells.



Inverted pyramid textured surface substantially reduces reflection losses and increases absorption probability in the device

Solar Cells Materials, Devices & Efficiencies

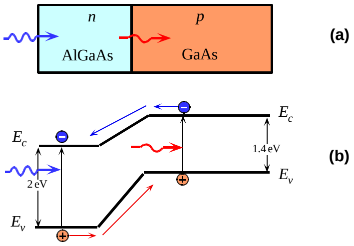
- Semiconductor alloy III-V → different bandgap with the same lattice constant → Heterojunction.



AlGaAs window layer on GaAs passivates the surface states and thereby increases the low wavelength photogeneration efficiency

Solar Cells Materials, Devices & Efficiencies

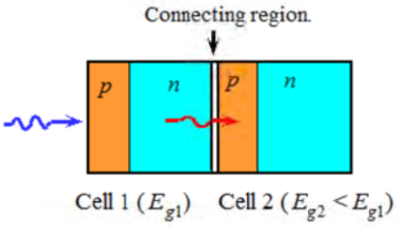
- Example *n*-AlGaAs with *p*-GaAs.



A heterojunction solar cell between two different bandgap semiconductors (GaAs and AlGaAs)

Solar Cells Materials, Devices & Efficiencies

- To further increase the absorbed photons → tandem or cascade cells (use two or more cells in tandem), such as GaAs – GaSb.



A tandem cell. Cell 1 has a wider bandgap and absorbs energetic photons with $h\upsilon > E_{g1}$. Cell 2 absorbs photons that pass cell 1 and have $h\upsilon > E_{g2}$.

CRYSTAL STRUCTURE OF IMPORTANT SEMICONDUCTORS

Silicon (Si), and Gallium Arsenide(GaAs)

CONTENTS

TERMINOLOGIES

CRYSTAL

- A crystal or crystalline is a solid substance in which atoms are highly arranged in ordered microscopic structure
- The word crystal is derived from Greek word “krustallos” meaning both ‘ice’ and ‘rock crystal’
- Structure of a crystal is a elucidation of the ordered arrangements of atoms, molecules or ions in a crystalline material

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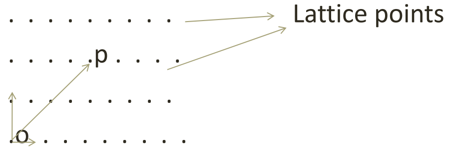
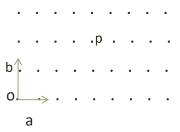
Lattice Points

- The points in the space, replacing the atoms in the structure of a crystal, with regular periodic arrangement and have identical environment with respect to other points, are called lattice points.



Primitive vectors

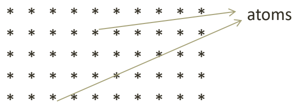
- The vectors which when repeated regularly give the array of lattice points in space lattice, they are known as *fundamental translational vectors* (or) *basis vectors* (or) *primitive vectors*.

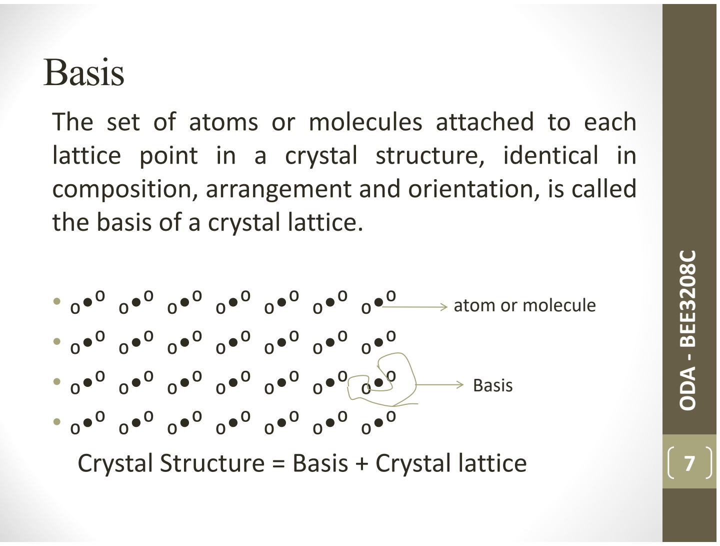


The position vector of the point p is given by $\bar{P} = 4\bar{a} + 2\bar{b}$

Space Lattice or Crystal Lattice

The regular orderly arrangement of lattice points in space which resembles the atoms or molecules in a crystal such that every point has same environment with respect to all other points is known as space lattice (or) crystal lattice.

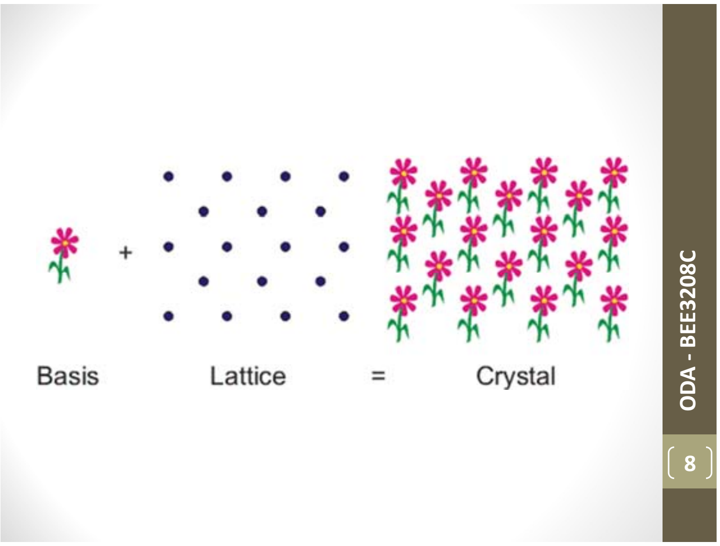


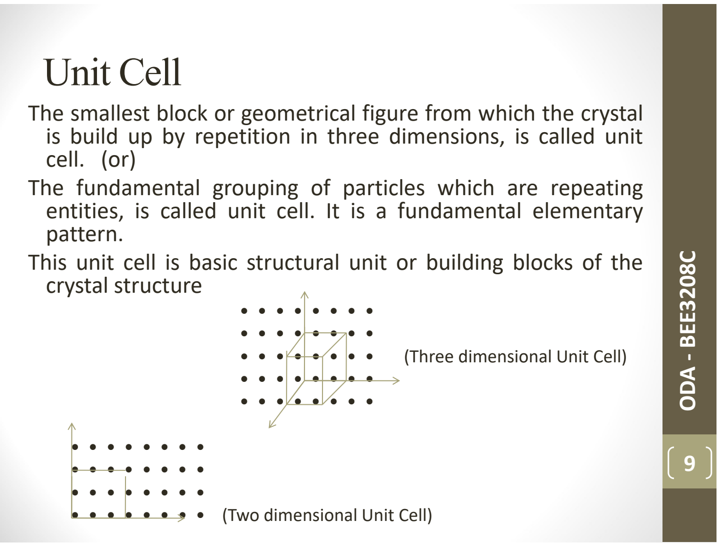


Basis

The set of atoms or molecules attached to each lattice point in a crystal structure, identical in composition, arrangement and orientation, is called the basis of a crystal lattice.

Crystal Structure = Basis + Crystal lattice





Unit Cell

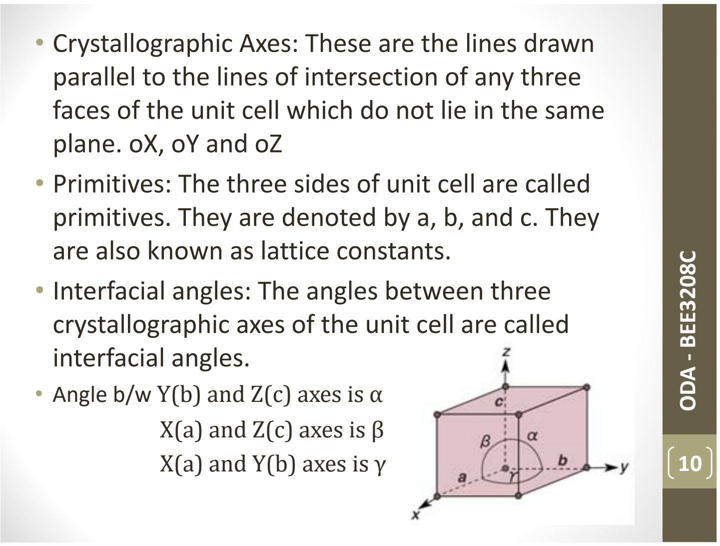
The smallest block or geometrical figure from which the crystal is build up by repetition in three dimensions, is called unit cell. (or)

The fundamental grouping of particles which are repeating entities, is called unit cell. It is a fundamental elementary pattern.

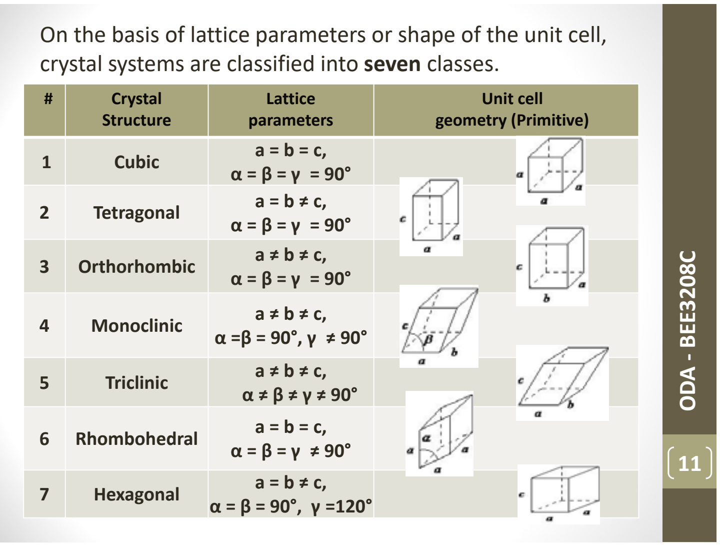
This unit cell is basic structural unit or building blocks of the crystal structure

(Three dimensional Unit Cell)

(Two dimensional Unit Cell)

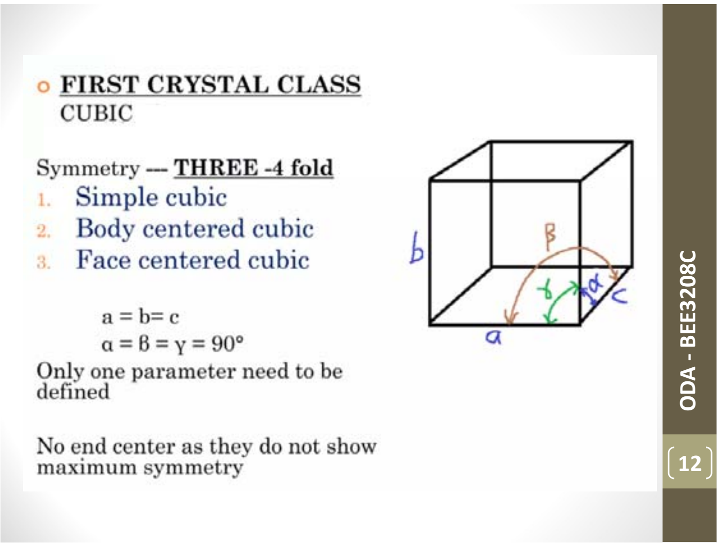


- Crystallographic Axes: These are the lines drawn parallel to the lines of intersection of any three faces of the unit cell which do not lie in the same plane. oX, oY and oZ
- Primitives: The three sides of unit cell are called primitives. They are denoted by a, b, and c. They are also known as lattice constants.
- Interfacial angles: The angles between three crystallographic axes of the unit cell are called interfacial angles.
- Angle b/w Y(b) and Z(c) axes is α
  X(a) and Z(c) axes is β
  X(a) and Y(b) axes is γ



On the basis of lattice parameters or shape of the unit cell, crystal systems are classified into **seven** classes.

#	Crystal Structure	Lattice parameters	Unit cell geometry (Primitive)
1	Cubic	$a = b = c$, $\alpha = \beta = \gamma = 90°$	
2	Tetragonal	$a = b \neq c$, $\alpha = \beta = \gamma = 90°$	
3	Orthorhombic	$a \neq b \neq c$, $\alpha = \beta = \gamma = 90°$	
4	Monoclinic	$a \neq b \neq c$, $\alpha = \beta = 90°, \gamma \neq 90°$	
5	Triclinic	$a \neq b \neq c$, $\alpha \neq \beta \neq \gamma \neq 90°$	
6	Rhombohedral	$a = b = c$, $\alpha = \beta = \gamma \neq 90°$	
7	Hexagonal	$a = b \neq c$, $\alpha = \beta = 90°, \gamma = 120°$	



- **FIRST CRYSTAL CLASS**
  CUBIC

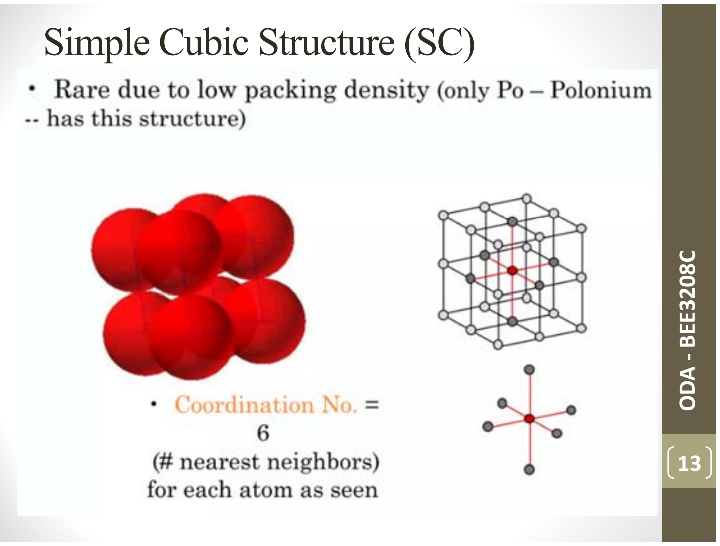
Symmetry --- **THREE -4 fold**

1. Simple cubic
2. Body centered cubic
3. Face centered cubic

$a = b = c$
$\alpha = \beta = \gamma = 90°$

Only one parameter need to be defined

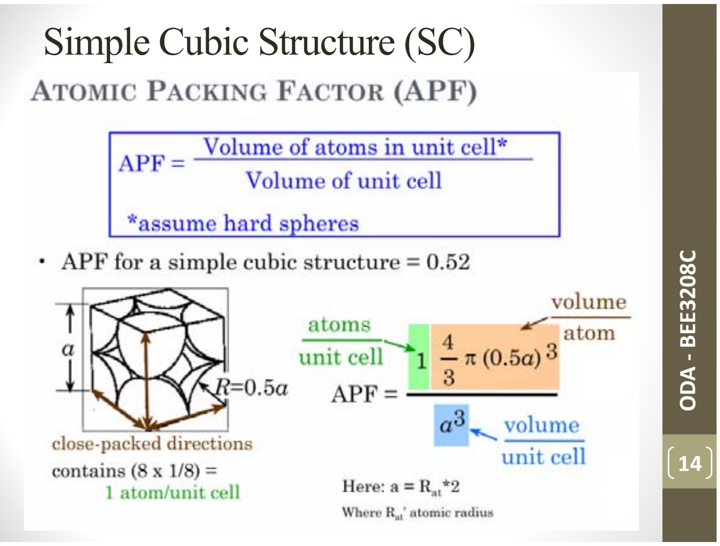
No end center as they do not show maximum symmetry



Simple Cubic Structure (SC)

- Rare due to low packing density (only Po – Polonium -- has this structure)

- Coordination No. = 6 (# nearest neighbors) for each atom as seen



Simple Cubic Structure (SC)

ATOMIC PACKING FACTOR (APF)

$$\boxed{\text{APF} = \frac{\text{Volume of atoms in unit cell*}}{\text{Volume of unit cell}}}$$

*assume hard spheres

- APF for a simple cubic structure = 0.52

close-packed directions

contains (8 x 1/8) = 1 atom/unit cell

$$\text{APF} = \frac{1 \ \frac{4}{3}\pi (0.5a)^3}{a^3}$$

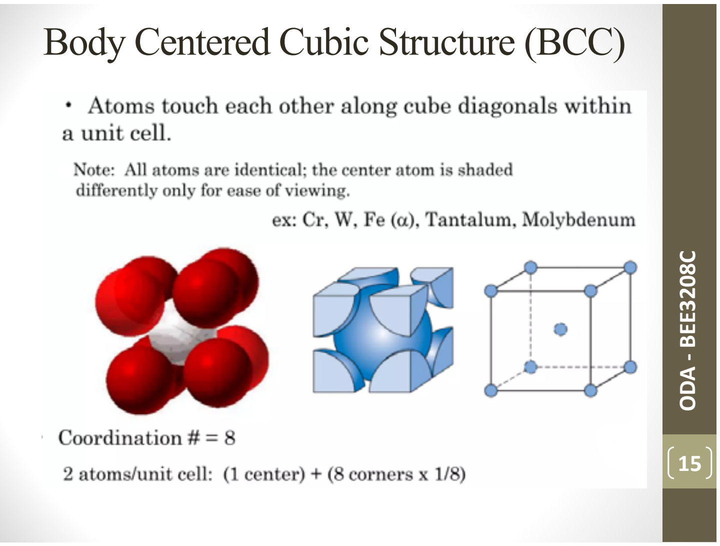
atoms/unit cell

volume/atom

volume/unit cell

Here: $a = R_{at}*2$

Where R_{at}' atomic radius



Body Centered Cubic Structure (BCC)

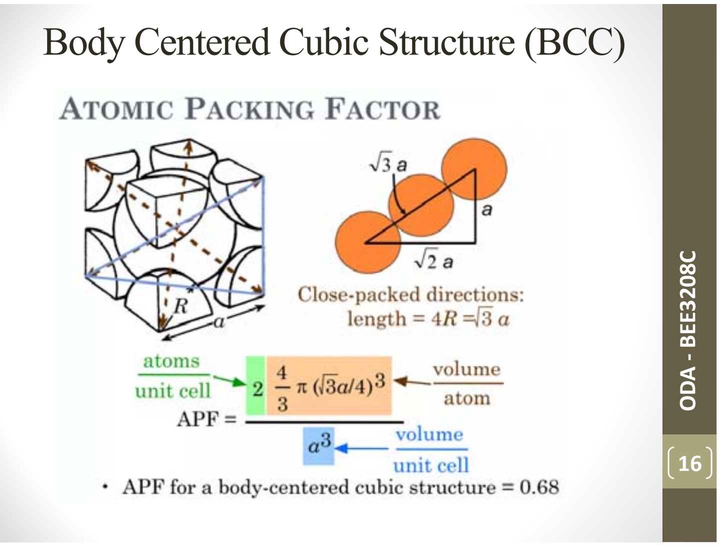
- Atoms touch each other along cube diagonals within a unit cell.

Note: All atoms are identical; the center atom is shaded differently only for ease of viewing.

ex: Cr, W, Fe (α), Tantalum, Molybdenum

Coordination # = 8

2 atoms/unit cell: (1 center) + (8 corners x 1/8)



Body Centered Cubic Structure (BCC)

ATOMIC PACKING FACTOR

Close-packed directions: length = $4R = \sqrt{3}\,a$

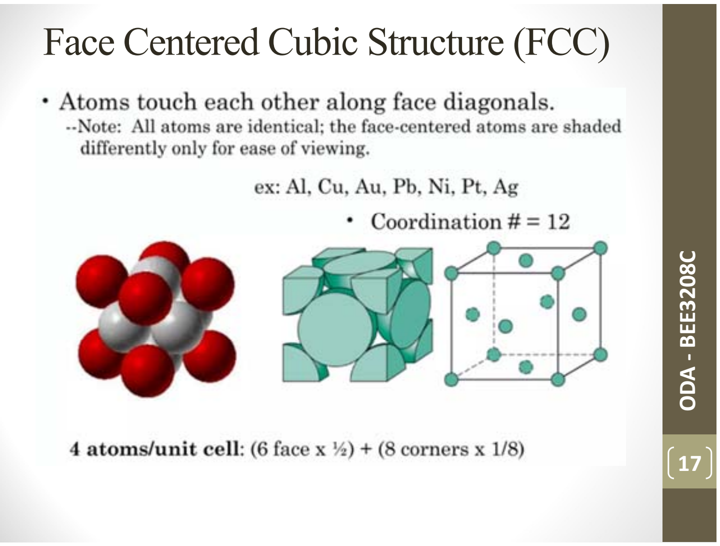
$$\text{APF} = \frac{2 \ \frac{4}{3}\pi (\sqrt{3}a/4)^3}{a^3}$$

atoms/unit cell

volume/atom

volume/unit cell

- APF for a body-centered cubic structure = 0.68



Face Centered Cubic Structure (FCC)

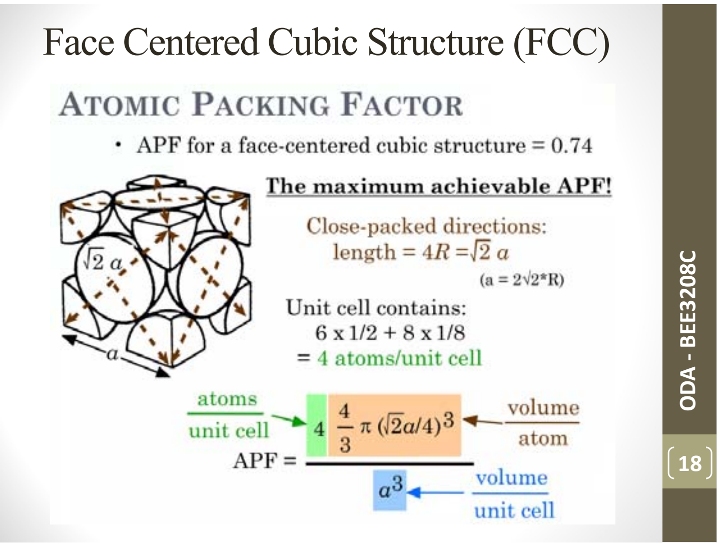
- Atoms touch each other along face diagonals.

--Note: All atoms are identical; the face-centered atoms are shaded differently only for ease of viewing.

ex: Al, Cu, Au, Pb, Ni, Pt, Ag

- Coordination # = 12

4 atoms/unit cell: (6 face x ½) + (8 corners x 1/8)



Face Centered Cubic Structure (FCC)

ATOMIC PACKING FACTOR

- APF for a face-centered cubic structure = 0.74

The maximum achievable APF!

Close-packed directions: length = $4R = \sqrt{2}\,a$

($a = 2\sqrt{2}*R$)

Unit cell contains:
6 x 1/2 + 8 x 1/8
= 4 atoms/unit cell

$$\text{APF} = \frac{4 \ \frac{4}{3}\pi (\sqrt{2}a/4)^3}{a^3}$$

atoms/unit cell

volume/atom

volume/unit cell

SECOND CRYSTAL CLASS
TETRAGONAL

- Symmetry– One 4 fold

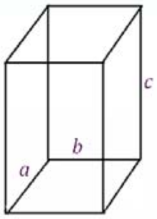
4…Simple tetragonal

5…Body centered tetragonal

$a = b$ not equal to c

$\alpha = \beta = \gamma = 90°$

- Two parameters to define a and c.
- **One might suppose stretching face-centered cubic would** result in face-centered tetragonal, but face-centered tetragonal is equivalent to body-centered tetragonal, BCT (with a smaller lattice spacing). BCT is considered more fundamental, so that is the standard terminology



Square Prism

THIRD CRYSTAL CLASS
ORTHORHOMBIC

- Symmetry–Three 2 fold

6…Simple

7…Body centered

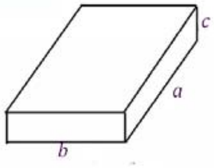
8…Face centered

9…End centered

$a \neq b \neq c$

$\alpha = \beta = \gamma = 90°$

- Three parameters to define *a*, *b* and *c*



Rectangle Prism

Fourth Crystal Class - Hexagonal

- Symmetry–One 6 fold

10…Simple Hexagonal

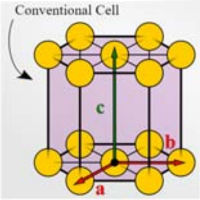
$a = b \neq c$

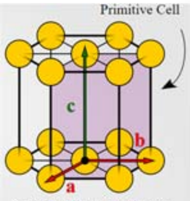
$\alpha = \beta = 90°$

$\gamma = 120°$

- Two parameters to define a and c.

The simple hexagonal (SH) unit cell can be imagined as a hexagonal prism with an atom on all 12 corners, and both faces. Simple Hexagonal has 3 atoms per unit cell (or 1 per primitive cell), lattice constant a = 2R, Coordination Number = 6, and APF = 61%.





Primitive cell accounts for one third of a conventional cell

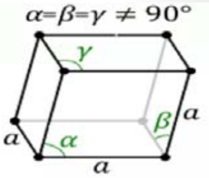
FIFTH CRYSTAL CLASS
Rhombohedral

11…Simple rhombohedral

Symmetry– One 3 fold (120°)

$a = b = c$

$\alpha = \beta = \gamma \neq 90°$



- Two parameters to define a and angle α.

SIXTH CRYSTAL CLASS
Monoclinic

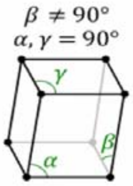
Symmetry– One 2 fold

12… Simple monoclinic

13…End centered Monoclinic (A and B not C)

$a \neq b \neq c$

$\alpha = \beta = 90° \neq \gamma$



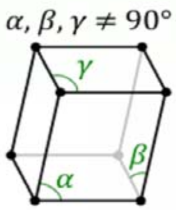
- Four parameters to define a ,b ,c and angle γ.

SEVENTH CRYSTAL CLASS
Triclinic

14… Simple triclinic

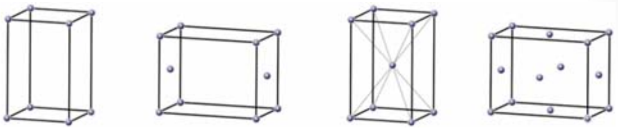
$a \neq b \neq c$

$\alpha \neq \beta \neq \gamma \neq 90°$



- Six parameters to define a ,b ,c and anglesα,β ,γ.
- HIGHLY UNSYMMETRIC CRYSTAL

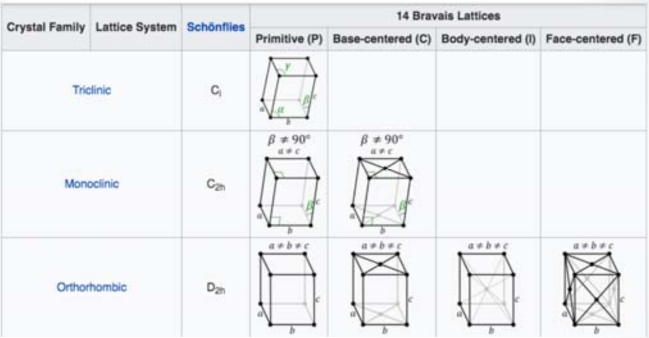
Summary of 4 types of cubic unit cells



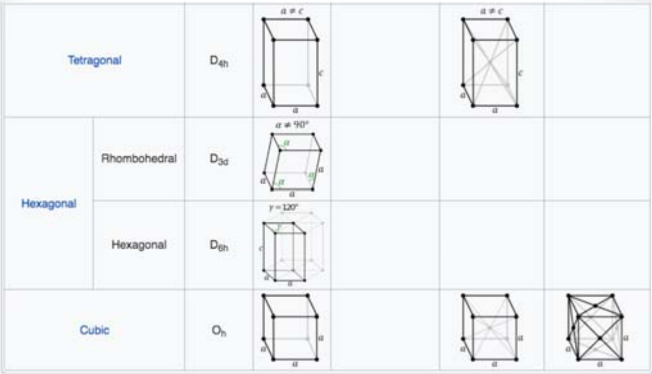
Primitive **P**	**Side or End-centered** **S or C(AB)**	**Body-centered** **I**	**Face-centered** **F**
One atom at each corner: 8 corners x 1/8 = **1 atom/unit cell**	One atom at each corner and 2 atoms at opposite faces 8 corners x 1/8 + 2x1/2 = **2 atoms/unit cell**	One atom at each corner and 1 atom in the middle of the cuboid 8 corners x 1/8 + 1 = **2 atoms/unit cell**	One atom at each corner and 1 atom in the middle of each face 8 corners x 1/8 + 6x1/2 = **4 atoms/unit cell**

Bravais Lattices

In 1948 Bravais showed that, there are fourteen ways of arranging points in space lattice, under the seven crystal classes to describe crystals. They are classified on the basis of the following crystal lattices.



Crystal Family	Lattice System	Schönflies	14 Bravais Lattices			
			Primitive (P)	Base-centered (C)	Body-centered (I)	Face-centered (F)
Triclinic		C_i				
Monoclinic		C_{2h}	$\beta \neq 90°$ $a \neq c$	$\beta \neq 90°$ $a \neq c$		
Orthorhombic		D_{2h}	$a \neq b \neq c$	$a \neq b \neq c$	$a \neq b \neq c$	$a \neq b \neq c$



Tetragonal		D_{4h}	$a \neq c$		$a \neq c$	
Hexagonal	Rhombohedral	D_{3d}	$\alpha \neq 90°$			
	Hexagonal	D_{6h}	$\gamma = 120°$			
Cubic		O_h				

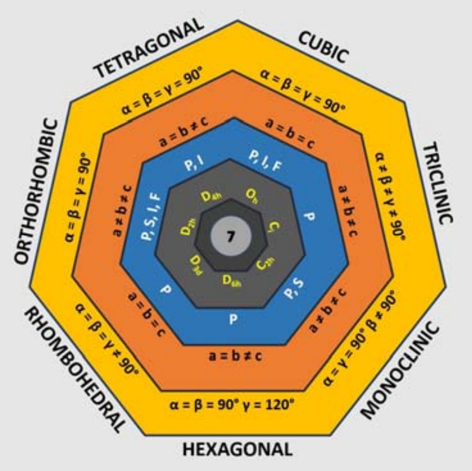
P = Primitive
I = Body Centred
F = Face Centred
S or C = Side/Base Centred

Primitive exists for all 7 classes

Body Centred exists in only 3 classes

Face Centred exists in only 2 classes

Side/Base Centred exists in only 2 classes



Expression for Lattice Constant

In a cubic unit cell, the sides of the cube are equal and constant, which is known as lattice constant.

a = b = c = lattice constant(a)

Consider a cubic unit cell with lattice parameters

a = b = c and

$\alpha = \beta = \gamma = 90^0$

Let ρ be the density of the cell.

Mass of the cubic unit cell $= a^3\ \rho$ ....(1)

If M is the molecular weight of the cell and N_A is the Avogadro's number,

Mass of each atom in unit cell = M / N_A

If n is number of atoms in a unit cell, then

Mass of the cubic unit cell $= nM / N_A$ ....(2)

from (1) and (2), $a^3\ \rho = nM / N_A$

$a^3 = nM / \rho N_A$

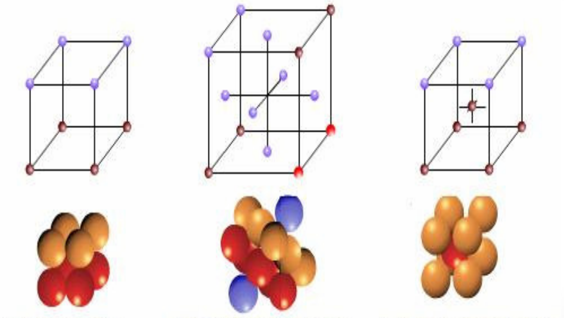
$$\boxed{a = [nM / \rho N_A]^{1/3}}$$

Important definitions

- **Atomic radius (r):** The half of the distance between any two successive atoms in a crystal lattice is called atomic radius.
- **Nearest Neighbour Distance (2r):** The distance between two nearest neighboring atoms in a crystal lattice is known as the nearest neighbour distance.
- **Effective number of atoms per unit cell:** The total number of atoms in a unit cell by considering the contribution of corner atoms, centre atoms and face centered atoms, is called Effective number of atoms per unit cell.
- **Coordination number (N):** The number of equidistant neighbors that an atom has in a crystal lattice is known as the coordination number.

Coordination Number

6 for SC 12 for FCC 8 for BCC



(a) Simple cubic (b) Face centred cubic (c) Body centred cubic

Three types of unit cells for cubic crystal system

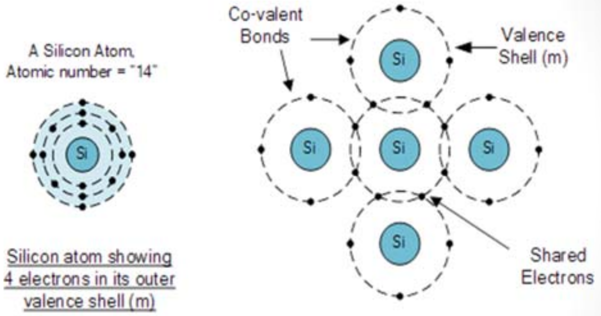
SILICON (Si)

- The name silicon is derived from Latin ‘silex’ or ‘silicis’ meaning ‘flint’ or ‘hard stone’
- Si is the second most abundant element on the earth crust
- It is first discovered by Swedish chemist Jons Jacob Berzelius in 1824 but in the form of amorphous element
- Crystalline form of silicon was not prepared until 1854, when it was obtained from electrolysis as a product

Si ATOM

- It is a non-metallic chemical element, which comes under carbon family (Group 14 in periodic table)
- Si has atomic number of 14. Each Si atom has 4 nearest neighbours

Si ATOMIC STRUCTURE

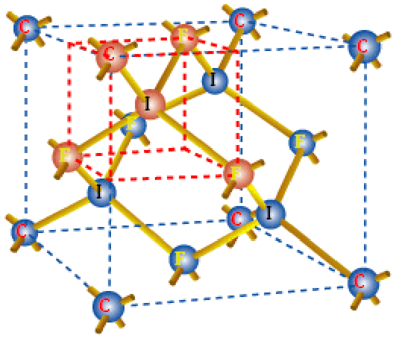


Silicon atom showing 4 electrons in its outer valence shell (m)

Silicon Crystal Lattice

SILICON CRYSTAL STRUCTURE

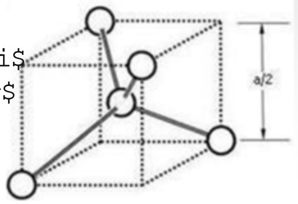
- Crystal structure is same as Diamond lattice, which has ‘2 interpenetrating FCC’ primitive lattice

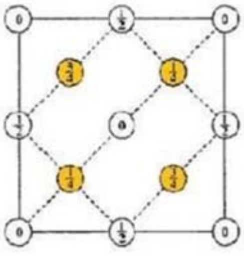


4 atoms are completely inside (I) the cell, 8 corner atoms (C) are shared among 8 cells so counting as 1 and the 6 atoms on the faces (F) are shared among 2 cells counting as 3.

So 4+1+3=8 atoms in unit cell

Xli$xixvelihvep$wxvygxyvi$
sj$gpswiwx$rimklfsyvwmr
xli$wmpmgsr$pexxmgi





Tvsnigxmsrsjexsqmg$evverkiqirx$
sr$e$gyfi$jegi$mr$e$wmpmgsr$yrmx$
gipp2$Jvegxmsrw$hirsxi$limklx$efszi$
xli$fewi$mr$yrmxw$sj$e$gyfi$ihki0$
xli$tsmrxw$ex4erh$˜$evisrxli$
JGG$pexxmgi?$xlswiexˆerh¯evi
sr$e$wmqmpev$pexxmgi$hmwtpegih$
epsrkxlifsh}$hmekserep$f}$ˆ$sj$
mxw$pirkxl2

The lattice constant of Silicon is 5.431Å or 0.5431 nm at 300K.

Hence, the atomic density or volume density of atoms can be calculated as follows:

Number of atoms per unit cell / volume of unit cell

$8 / (5.431\times10^{-10})^3$ = **5×10^{28} atoms/m³** or **5×10^{22} atoms/cm³**

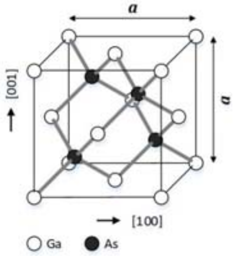
Because of its structure, silicon is the most important semiconductor in the electronic and technology sector.

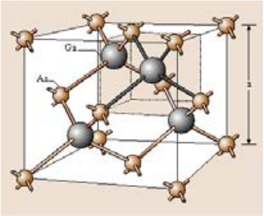
GALLIUM ARSENIDE

- Gallium arsenide is a combination of one Gallium atom and one Arsenic atom. Gallium was discovered by Lecoq de Boisbaudran in 1875 at Pyrenees Mountains
- Arsenic was discovered in 1230 by Albertus Magnus and it was later found to be poisonous
- Later in 1954 Victor Goldschmidt combined these two elements
- Since, then it is used in many devices such as LEDs, etc.,

Gallium Arsenide Crystal Structure

- Gallium is a poor metal and its located at Group 13 in periodic table
- Arsenic is a metalloid and its located at Group 15 in periodic table





GaAs is a compound, each gallium atom is surrounded by four arsenic atoms and vice versa. Coordination geometry is tetrahedral similar to Silicon.

- Atomic number of Gallium is 31 and for Arsenic its is 33
- GaAs crystal is composed of two sub-lattices. Each sub-lattices are FCC lattice. The lattice constant is **5.65Å**
- They are arranged with respect to each other by half the diagonal of a unit cell.
- Atomic density is around **4.45×10^{22} atoms/cm³**
- The crystal structure is known as zinc blende or cubic sphalerite

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Difference between Diamond and Zinc Blende structure

Feature	Silicon (Diamond Structure)	Zinc Blende (Sphalerite)
Material Type	Elemental (e.g., C, Si, Ge)	Binary compound (e.g., ZnS, GaAs, SiC)
Basis Atoms	Two identical atoms	Two different atoms
Lattice Type	Face-centered cubic (FCC)	Face-centered cubic (FCC)
Coordination Number	4	4
Sublattices	Two interpenetrating identical FCC lattices	Two interpenetrating different FCC lattices
Typical Bandgap	Moderate (Semiconductor)	Varies (often wide, e.g., ZnS)

SUMMARY

- Depending on the conductivity materials are divided into conductors, insulators and semiconductors
- Silicon is the most commonly used semiconductor
- Crystals are the ordered arrangement of atoms in a material
- The development of semiconductors are essential for modern electronics
- Silicon has a diamond lattice structure whereas Gallium Arsenide has a Zinc Blende lattice structure

FIBER OPTICS

CONTENTS

- ➢OPTICAL FIBRE
- ➢TYPES OF OPTICAL FIBRE
  - Single Mode (SM)
  - Multi Mode (MM)
  - V-number / Normalized Frequency
- ➢ REFRACTION AND INDEX OF REFRACTION
  - Snell's Law
  - Total Internal Reflection
- ➢ LIGHT ENTERING AN OPTICAL FIBRE
  - ▪ Acceptance Angle and Cone
  - ▪ Numerical Aperture (NA)

Optical Fibre

- An optical fibre is a dielectric channel used to guide light by the principle of Total Internal Reflection (TIR)
- An optical fibre is a flexible*, transparent fibre made by drawing glass (silica) or plastic to a diameter slightly thicker than that of a human hair.
- Optical fibres are used most often as a means to transmit light between the two ends of the fibre and find wide usage in fibre-optic communications

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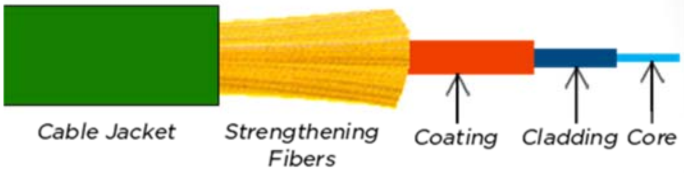
Basic Optical Fibre Construction



A fibre optic cable consists of five main components: core, cladding, coating, strengthening fibres, and cable jacket.

- **Core:** This is the physical medium that transports optical signals from an attached light source to a receiving device. The core is a single continuous strand of high-purity glass or plastic whose diameter is measured in microns (less than the diameter of a human hair). The larger the core, the more light the cable can carry, which means to a higher data transfer rate.

- **Cladding:** This is a thin layer that is extruded over the core and serves as the boundary that contains the light waves, enabling data to travel through the length of the fibre.

- **Coating:** This is a plastic coating over the cladding to reinforce the fibre core, help absorb shocks, and provide extra protection against excessive cable bends. It does not have any effect on the optical waveguide properties, though.

- **Strengthening fibres:** These components help protect the core against crushing forces and excessive tension during installation. The materials can range from Kevlar, to wire strands, to gel-filled sleeves. Sometimes light-absorbing glass is added between the fibres, to prevent light leaking out of one fibre from entering another. This reduces cross-talk between the fibres.

- **Cable Jacket:** This is the outer layer, or sheathing, of the cable. Its purpose is to protect the cable from environmental hazards, such as construction work, fishing gear, and even sharks, which are often attracted to the electrical fields created by signal conductors to repeaters.

Types of optical fibre

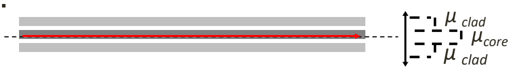
Based on their transmission properties and structure, there are of two categories:

1. **Mono or Single-Mode Fibre (SM)**

2. **Multi-mode Fibre (MF)**
   a) **Step Index Fibre**
   b) **Graded Index Fibre**

Mono or Single Mode

This is called so because the refractive index of the fibre ‘**step**’ up as we move from the cladding to the core and this type of fibre allows single mode to propagate at a time due to very small diameter of its core.



The refractive indices of the cladding and the core remain constant

The size of its core (diameter) is typically around 9-10 μm.

Multi-Mode (MM)

This is called so because it allow more than one mode to propagate. Over more than 100 modes can propagate through multimode fibres at a time. The size of its core is typically around 50 μm or 62.5 μm.

Two Types

- **Step Index Fibres**
- **Graded Index Fibres**

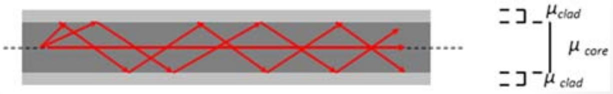
Multi-Mode Step Index

No. of propagating modes α Core diameter/ Wavelength

Typically the core diameter is 50 to 100 μm and Numerical Aperture (NA) varies from 0.20 to 0.29 respectively

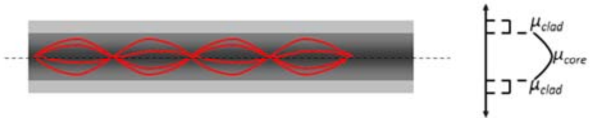
Due to higher value of NA , and larger core size in this case, fibre connections and injection of light is very easy.

Due to several modes, the effect of dispersion gets increased, i.e. the modes arrive at the fibre end slightly different times and so spreading of pulses takes place.



Multi-Mode Graded Index

In this fibre, the refractive index of the core decreases with increasing radial distance from the fibre axis.
The value of the refractive index is highest at the centre of the core and decreases to a value at the edge of the core that equal the refractive index of the cladding.



In this type of fibre design, the dispersion of the modes is compensated. Also, light wave follow sinusoidal paths along the fibre.

Multi-Mode Graded Index

The profile of the refractive index is nearly parabolic that results in continual refocusing of the ray in the core, and minimizing the model dispersion.

Standard graded index fibres typically have a core diameter of 50 μm or 62.5 μm and the cladding diameter of 125 μm.

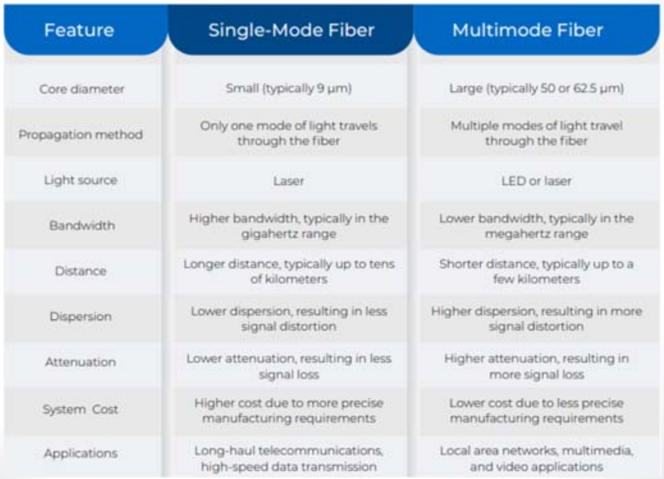
Advantage of Graded Index Fibre over Step Index Fibre

Decrease in the modal dispersion

Advantage of SM over MM fibres

- **Lower signal loss and a higher information capacity or bandwidth than multi-mode fibres as the signal loss depends on the operational wavelength**
- **SM fibres are capable of transferring higher amount of data due to low fibre dispersion**
- **SM fibres are known as low loss fibres**

Summary of optical fibres



Feature	Single-Mode Fiber	Multimode Fiber
Core diameter	Small (typically 9 µm)	Large (typically 50 or 62.5 µm)
Propagation method	Only one mode of light travels through the fiber	Multiple modes of light travel through the fiber
Light source	Laser	LED or laser
Bandwidth	Higher bandwidth, typically in the gigahertz range	Lower bandwidth, typically in the megahertz range
Distance	Longer distance, typically up to tens of kilometers	Shorter distance, typically up to a few kilometers
Dispersion	Lower dispersion, resulting in less signal distortion	Higher dispersion, resulting in more signal distortion
Attenuation	Lower attenuation, resulting in less signal loss	Higher attenuation, resulting in more signal loss
System Cost	Higher cost due to more precise manufacturing requirements	Lower cost due to less precise manufacturing requirements
Applications	Long-haul telecommunications, high-speed data transmission	Local area networks, multimedia, and video applications

Maximum Allowed Modes

Maximum Number of modes that propagate successively in the fibre

$$m_m = \frac{1}{2}\left(\pi \frac{d}{\lambda} NA\right)^2$$

$$m_m \propto \frac{d}{\lambda}$$

Hence, number of possible modes will be larger for higher ratio d/λ

For multimode fibres $m_m > 2$

For single mode fibres $m_m < 2$

$$\frac{d}{\lambda} < \frac{2}{\pi NA}$$

As is evident the parameter m_m decides the number of possible modes since this parameter depends on core diameter d and the numerical aperture NA. Therefore, the number of allowed modes would be different for fibres of different core diameters.

V-number or Normalized Frequency

V number: determines how many modes a fiber supports

$$V = \frac{2\pi a}{\lambda}\sqrt{\mu^2 - \mu^2} = \frac{2\pi a}{\lambda} NA$$

For SM fiber, the V number is =< 2.405, it is greater for MM

Normalized Frequency ➡

$$\nu_n = \pi \frac{d}{\lambda} NA = \pi \frac{d}{\lambda}\sqrt{\mu_1^2 - \mu_2^2}$$

Therefore, the number of modes m_m in terms of normalized frequency is:

$$m_m = \frac{\nu_n^2}{2}$$

For Step-Index

$$m_m = \frac{\nu_n^2}{4 \text{ or } 6}$$

For Graded-Index

Angular Coupling Efficiency (η_{ang})

Angular Coupling Efficiency ≈ NA^2

Hence, Input Coupled Power = NA^2 x Output Power

Assumptions:

1. Source is smaller compared to fibre core
2. Source is a Lambertian emitter such as a small, planar, Surface LED.
3. Fresnel reflection is assumed to be negligible
4. Suited to Multimode Step Index Fibre.

Worked Example

Question:

A multimode step index fibre with core diameter 50 µm and relative refractive index of 1.4% is operating at 850 nm. The core refractive index is 1.45.

i) Determine the V-parameter for the fibre.

ii) What is the power coupled into the fibre if the cladding refractive index is 1.41 assuming a LED radiates 70 µW of power?

Solution:

We first calculate NA

Relative refractive index (Δ) = (n1-n2)/n1 = (1.45-n2)/1.45 = 1.4/100 = 0.014

n2 = 1-45 – (1.45x0.014) = 1.4297

NA = $(1.45^2 - 1.4297^2)^{0.5}$ = 0.2418

i) V-parameter = (π.d/ λ) * NA = (π * 50x10^{-6} / 850x10^{-9})* 0.2418 = **44.68**

ii) NA = $(1.45^2 - 1.41^2)^{0.5}$ = 0.338

Max. Coupling Efficiency = NA^2 = 0.338^2 = 0.1142

Input power coupled = 0.1142*70 = **8 µW**

Refraction and Index of Refraction

When light moves for one transparent medium to another, its velocity changes. The denser the medium, the slower its velocity. This change in velocity manifest itself as a ‘bending effect’ called **refraction**. Media of interest to us air and glass (silica) or plastic.
The index of refraction or refractive index of a medium (n) can be calculated as follows:

index of refraction

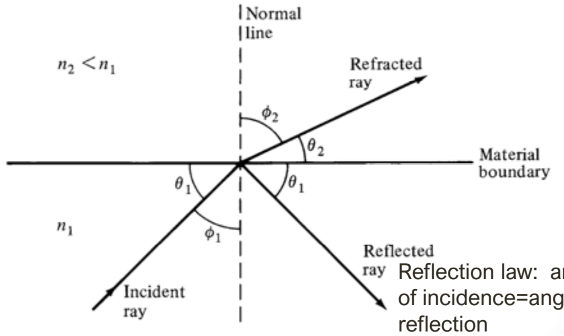
velocity of light in vacuum

$$n = \frac{c}{v}$$

velocity of light in the medium

Laws of reflection & refraction

When a ray is incident on the interface between two dielectrics of different refractive indices (e.g. glass-air), reflection and refraction both occur.



Reflection law: angle of incidence=angle of reflection

Snell’s law of Refraction: $n_1 \sin\phi_1 = n_2 \sin\phi_2$

Laws of reflection & refraction

- If n2 < n1 , then the angle of refraction is greater than the angle of incidence and the refracted ray is said to have moved away from the normal.
- If the angle of incidence (ϕ_1) is increased further, the angle of refraction (ϕ_2) also increases in accordance with the Snell’s law and at a particular angle of incidence the angle of refraction becomes 90^o and the refracted ray grazes along the media interface.
- This angle of incidence is called the **critical angle of incidence (θc)** of medium 2 with respect to medium 1. One should note here that critical angle is media-relative.
- If (ϕ_1) is increased beyond the critical angle, there exists no refracted ray and the incident light ray is then reflected back into the same medium. This phenomenon is called the **total internal reflection** of light.

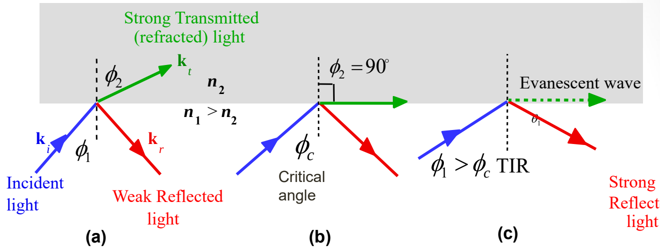
Total Internal Reflection (TIR)

- The word ‘total’ signifies that the entire light energy that was incident on the media interface is reflected back into the same medium. Total Internal Reflection (TIR) obeys the laws of reflection of light.
- This phenomenon shows that light energy can be made to remain confined in the same medium.

Thus we can see that there are two basic requirements for a TIR to occur:

1. The medium from which light is incident, must be optically denser than the other medium. i.e n1 > n2 e.g. core and cladding.
2. The angle of incidence in the denser medium must be greater than the critical angle.

Total Internal Reflection (TIR)

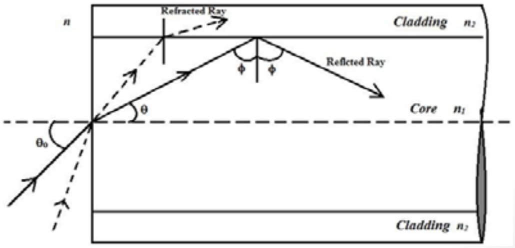


$\phi_1 < \phi_c$ $\phi_1 = \phi_c$ $\phi_1 > \phi_c$

$n_1 \sin\phi_1 = n_2 \sin\phi_2$ $\sin\phi_c = \frac{n_2}{n_1}$

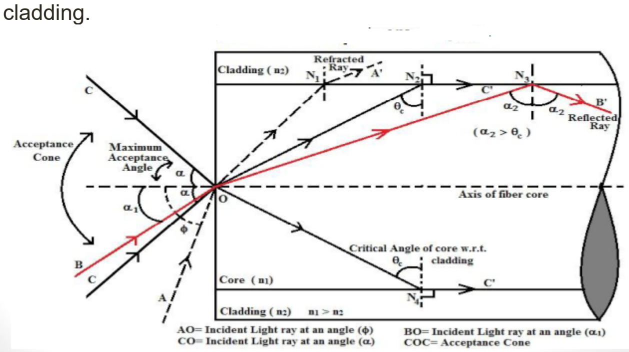
Light entering an optical fiber

- Light propagates inside an optical fiber by virtue of multiple TIRs at the core-cladding interface.
- The refractive index of the core glass is greater than that of the cladding. This meets the first condition for a TIR.
- All the light energy that is launched into the optical fiber through its tip does not get guided along the fiber. Only the light rays which are launched into the fiber at such an angle that the refracted ray inside the core of the fiber is incident on the core-cladding interface at an angle greater than the critical angle of the core with respect to the cladding.



Acceptance Angle and Cone

- Assuming that light is launched as meridional rays into the optical fibre,
- The incident ray AO (shown by dotted line) is incident at an angle ϕ with the axis of the fibre. The refracted ray for AO in the core (dotted line ON1) fails to be incident on the core-cladding interface at angle greater or equal to the critical angle and hence refracts out of the core and is lost to the cladding.

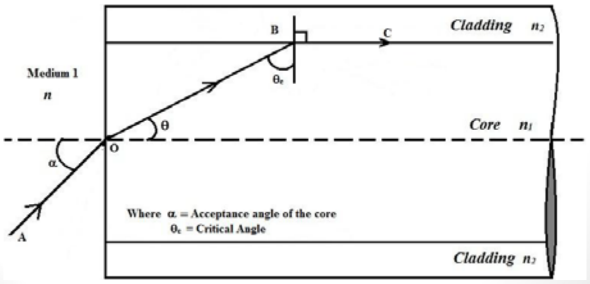


Acceptance Angle and Cone

- The ray CO is launched into the fiber at such an angle 'α' that its refracted ray is incident at the core-cladding boundary at its critical angle 'θc'.
- If any light ray is launched at an Fiber Optic angle more than α then the refracted ray just refracts out to the cladding because the angle of incidence of its refracted ray at the core-cladding interface is less than the critical angle.
- Thus the angle α is indicative of the maximum possible angle of launching of a light ray that is accepted by the fiber.
- Consequently, the angle α is called the **angle of acceptance** of the fiber core.
- Since the optical fiber is symmetrical about its axis, it is very clear that all the launched rays, which make an angle α with the axis, considered together, form a sort of a cone.
- This cone is called the **acceptance cone**.

Numerical Aperture (N.A)

- An incident ray AO is incident from medium1 at the tip of the fiber making an angle α with the axis of the fiber, which is the acceptance angle of the fibre.
- The refracted ray for this incident ray in the core then is incident at the core-cladding interface at the critical angle θc of the core with respect to the cladding. The angle of refraction for critical angle of incidence is 90^0 and the refracted ray thus grazes along the core-cladding boundary along BC as shown below:



Numerical Aperture (N.A)

According Snell's law, the incident and the refracted rays lie in the same meridional plane, which is the plane of the paper in this case. Applying Snell's law at the medium1-core interface we get:

$$n\sin\alpha = n_1\sin\theta \qquad (2.1)$$

From the figure it is clear that, $\theta = \frac{\pi}{2} - \theta_c$ substituting this in equation (2.1), we get:

$$n\sin\alpha = n_1\cos\theta_c \qquad (2.2)$$

From the basic trigonometric ratios, $\cos\theta_c = \sqrt{1-\sin^2\theta_c}$ (2.3)

Applying Snell's law at the core-cladding interface we get: $\sin\theta_c = \frac{n_2}{n_1}$

Numerical Aperture (N.A)

$$\cos\theta_c = \sqrt{1-\left(\frac{n_2}{n_1}\right)^2} \qquad (2.4)$$

Substituting equation (2.4) in equation (2.2) we get: $n\sin\alpha = n_1\cos\theta_c$

$$n\sin\alpha = \sqrt{n_1^2 - n_2^2}$$

- Since the initial medium 1 from which the light is launched is air most of the times, n = 1. The angle α is indicative of light accepting capability of the optical fiber.
- The greater the value of α, more light accepted by the optical fiber. In other words, the optical fiber acts as some kind of aperture that accepts only some amount of the total light energy incident on it. The light accepting efficiency of this aperture is thus indicated by **sin α and hence this quantity is called as the numerical aperture (N.A.)**

$$N.A = \sin\alpha = \sqrt{n_1^2 - n_2^2} \qquad (2.5)$$

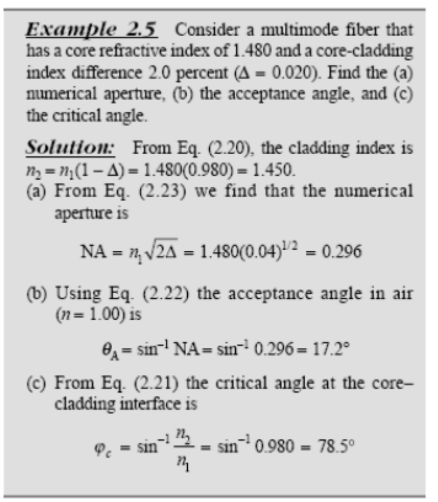
Worked Example

relative refractive index Δ

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$$NA \cong n_1\sqrt{2\Delta}$$

$$N.A = \sin\alpha$$



Example 2.5 Consider a multimode fiber that has a core refractive index of 1.480 and a core-cladding index difference 2.0 percent ($\Delta = 0.020$). Find the (a) numerical aperture, (b) the acceptance angle, and (c) the critical angle.

Solution: From Eq. (2.20), the cladding index is $n_2 = n_1(1-\Delta) = 1.480(0.980) = 1.450$.

(a) From Eq. (2.23) we find that the numerical aperture is

$$\text{NA} = n_1\sqrt{2\Delta} = 1.480(0.04)^{1/2} = 0.296$$

(b) Using Eq. (2.22) the acceptance angle in air ($n = 1.00$) is

$$\theta_A = \sin^{-1}\text{NA} = \sin^{-1}0.296 = 17.2°$$

(c) From Eq. (2.21) the critical angle at the core–cladding interface is

$$\varphi_c = \sin^{-1}\frac{n_2}{n_1} = \sin^{-1}0.980 = 78.5°$$

UNIVERSITY of TECHNOLOGY, MAURITIUS

B.Eng (Hons.) Electronic Engineering

Cohort: BEE/09/FT

Examinations for 2012 – 2013 / Semester 2

MODULE: **OPTOELECTRONICS**
MODULE CODE: ELEC4105
Duration: 2½ Hours
Reading time: None

Instructions to Candidates:

1. Attempt ALL **FOUR questions**.
2. Start your answer to each question on a fresh page.
3. All questions carry **equal** marks.
4. Maximum Marks = **100**
5. Silent calculators are allowed in the Examination Room.
6. Appendix is provided.

This question paper contains 4 questions and 5 pages.

ATTEMPT ALL FOUR QUESTIONS

QUESTION 1: (25 MARKS)

a) Describe **two** mechanisms by which electrons can be excited from the valence band to the conduction band?

(4 marks)

b) Given that the direct-band gap energy for Gallium Nitride (***GaN***) is **3.4 eV** at room temperature, calculate the phonon energy and phonon momentum that you can expect from such a material with a lattice constant of **3.186 Å** and velocity of sound of **$5x10^3$ m/s**.

(3+2 marks)

c) Given a **P-type** semiconductor with **N_A** and **N_D** atom concentrations of **$8x10^{11}$ cm^{-3}** and **$3x10^3$ cm^{-3}** respectively, calculate the electron concentration assuming **$n_i = 1.4x10^{10}$ cm^{-3}**.

(3 marks)

d) What do you understand by the Fermi Level of a crystal? According to the Fermi-Dirac function, what is the temperature at which there is **1%** probability that a state, with an energy **0.5 eV** above the Fermi energy, will be occupied by an electron?

(7 marks)

e) Given a piece of **N-type** silicon at **0 ºC**,

   i. Calculate the thermal velocity of an electron with an effective mass of **$1.18m_0$** and a mobility of **0.25 $m^2V^{-1}s^{-1}$**.
   ii. What is the electric field that needs to be applied to make the electron drift at a velocity of **1000 ms^{-1}**?

(4+2 marks)

QUESTION 2: (25 Marks)

a) Describe **five** advantages of LED lighting over conventional incandescent lighting.

(5 marks)

b) State **two** impairments that reduces the radiant flux of a surface-emitting LED. Illustrate one way how any one of these impairments may be addressed?

(4 marks)

c) Illustrate how white light can be generated using a blue Gallium Nitride *(GaN)* LED.

(3 marks)

d) Find the composition of the quaternary alloy $In_{1-x}Ga_xAs_yP_{1-y}$ for making semiconductor LEDs operating at **1.27μm**. The bandgap energy Eg of the quartenary alloy in eV is then given by the empirical relationship $E_g = 1.35 - 0.72y + 0.12y^2$ where $y=2.2x$.

(6 marks)

e) Given a particular Gallium Nitride *(GaN)* Edge-emitting LED drawing a current **50 mA** when the voltage is **1.5 V** and operating at **450 nm** wavelength with light power of **5 μW** coupled with a **9 μm** single-mode fibre.

- i) What is the colour of the light emitted from the LED?
- ii) Calculate the overall efficiency of the LED.
- iii) Calculate its internal quantum efficiency given that $T_r = 50ns$ and $T_{nr} = 70\ ns$.
- iv) Hence, calculate the internal optical power generated by the LED.

(1+2+1+3 marks)

QUESTION 3: (25 Marks)

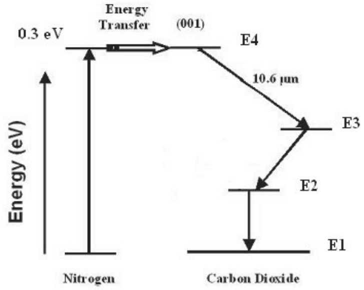
a) State **four** general properties of lasers.

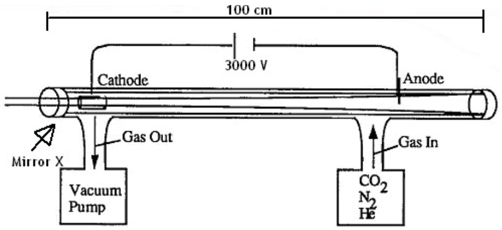
(4 marks)

b) Describe why '*population inversion*' is a necessary condition for sustained lasing action?

(4 marks)

c) Consider a **400 mW** nitrogen-carbon dioxide laser operating at **10.6 μm**. The operating temperature is **350 °C** and the current is **20 mA**. *(Mass of CO_2 molecule= $7.16x10^{-26}$ kg)*





- i) Calculate the energy value at level E3 in eV and state which level is meta-stable?
- ii) Describe Mirror X in the diagram above and calculate the efficiency of the laser?
- iii) Estimate the Doppler broadened linewidth (Δv) in the output spectrum.
- iv) What are the mode number m values that satisfy the resonant cavity condition?
- v) What are the mode separations in frequency (Δv_m) and wavelength ($\Delta\lambda_m$)?

(4+3+4+2+4 marks)

QUESTION 4: (25 Marks)

a) State **six** desirable requirements of a generic photo-detector.

(6 marks)

b) A gallium arsenide *pin* photo-diode has an intrinsic layer of width **1 µm**. The diode is reverse-biased by a voltage of **3 V** and then illuminated with short optical pulse of wavelength **800 nm**.

   i) Deduce the penetration depth to know if absorption will occur across the whole width.

   ii) Calculate the electric field across the intrinsic layer width.

   iii) Estimate the electron drift velocity.

   iv) Estimate the duration of the photo-current across the whole width of the diode.

(4x2 marks)

c) A photo-diode has an active circular area with radius **2 mm**. When radiation of wavelength **800 nm** and intensity **0.25 mW/cm^2** is incident it generates a photo-current of **0.3 µA**. What is the responsivity and quantum efficiency of the photo-diode at **800 nm**?

(4+2 marks)

d) A **0.5 cm^2** photo-detector is shone with **850 nm** light of intensity **75 mW/cm^2**. Assume that each photon generates one EHP, calculate the number of EH pairs created per second.

(3 marks)

e) What is the thickness of a Germanium crystal layer that is needed for absorbing **85%** of the incident radiation at **1 µm**?

(2 marks)

******END OF EXAM PAPER******

UNIVERSITY of TECHNOLOGY, MAURITIUS

B.Eng (Hons.) Electronic Engineering

Cohort: BEE/10A/FT

Examinations for 2013 / Semester 2

MODULE: OPTOELECTRONICS
MODULE CODE: ELEC4105
Duration: 2½ Hours
Reading time: None

Instructions to Candidates:

- Attempt ALL **FOUR questions**.
- Start your answer to each question on a fresh page.
- All questions carry **equal** marks.
- Maximum Marks = **100**
- Silent calculators are allowed in the Examination Room.
- Appendix provided

This question paper contains 4 questions and 5 pages.

ATTEMPT ALL FOUR QUESTIONS

QUESTION 1: (25 MARKS)

a) Describe three mechanisms by which electrons can be excited from the valence band to the conduction band?

(6 marks)

b) Given that the band gap energy for Indium Phosphide *(InP)* is 1.344 eV at 300K, calculate the photon momentum that you can expect from such a material.

(3 marks)

c) Given a N-type semiconductor with $\mathbf{N_A}$ and $\mathbf{N_D}$ concentrations of $1\text{x}10^7$ atoms cm^{-3} and $3\text{x}10^{10}\ \text{cm}^{-3}$ respectively, calculate the electron concentration assuming $n_i = 1.4\text{x}10^{10}\ \text{cm}^{-3}$.

(3 marks)

d) What do you understand by the Fermi Level of a crystal? According to the Fermi-Dirac function, at what temperature can we expect a 40% probability that electrons in Gold will have an energy which is 5% above the Fermi level? (E_F (Gold) = 5.5 eV)

(6 marks)

e) An electron is moving in a piece of lightly doped silicon under an applied field at 27 °C so that its drift velocity is one-tenth of its thermal velocity.

   i. Calculate the average number of collisions it will experience while drifting a region 1μm wide given $m_e^* = 0.26m_0$ and electron mobility = $0.1414\ \text{m}^2\text{V}^{-1}\text{s}^{-1}$.
   ii. What is the applied voltage across this region?

(5+2 marks)

QUESTION 2: (25 Marks)

a) Describe **five** advantages of LED lighting over conventional incandescent lighting.

(5 marks)

b) Describe **two** ways to reduce loss of radiant flux in an LED due to total internal reflection of photons at the output interface.

(4 marks)

c) Explain how white light can be generated using a UV LED.

(3 marks)

d) Find the composition of the quaternary alloy $In_{1-x}Ga_xAs_{1-y}P_y$ for making semiconductor LEDs operating at 1.5μm. The bandgap energy Eg of the quartenary alloy in eV is then given by the empirical relationship $Eg = 1.35 - 0.72y + 0.12y^2$ where $x=0.5y$.

(6 marks)

e) Given a particular Gallium Nitride *(GaN)* LED drawing a current 75 mA when the voltage is 3 V with an external conversion efficiency of 10% and operating at 400 nm wavelength.

   i) What is the colour of the light emitted from the LED?
   ii) Calculate its external optical power output.
   iii) Calculate its internal quantum efficiency given that $T_r = 35ns$ and $T_{nr} = 50\ ns$.
   iv) Hence, calculate the internal optical power generated by the LED.

(1+2+1+3 marks)

QUESTION 3: (25 Marks)

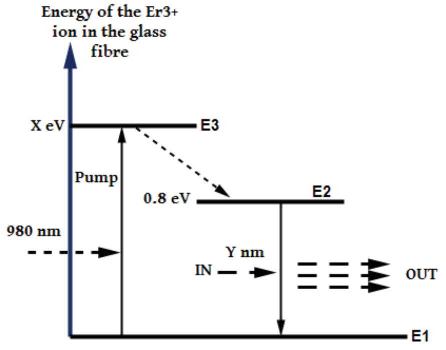
a) State **four** general properties of lasers.

(4 marks)

b) Why is 'population inversion' necessary for sustained lasing action?

(4 marks)

c) Consider a hypothetical Er3$^+$ doped optical fibre amplifier (EDFA) below:



- i) Calculate the value of X in eV at level E3.
- ii) Describe what happens between level E3 and E2 as shown by the dotted arrow?
- iii) Calculate the value of Y in nm between level E2 and E1?

(2+4+2 marks)

d) A particular 2 Watt Krypton laser operating at 400 nm has a tube that is 30 cm long. The operating temperature is at 400 K at a voltage of 110 V drawing a current of of 10 A. (Mass of Krypton atom = 1.39x10^{-25} kg)

- i) What is the efficiency of the laser?
- ii) Estimate the Doppler broadened linewidth (Δv) in the output spectrum.
- iii) What are the mode number m values that satisfy the resonant cavity condition?
- iv) What are the mode separations in frequency (Δv_m) and wavelength ($\Delta \lambda_m$)?

(1+3+1+4 marks)

QUESTION 4: (25 Marks)

a) State **five** desirable requirements of a generic photo-detector.

(5 marks)

b) Describe **two** drawbacks that are addressed by **pin** photo-diode compared to conventional **pn** junction photo-diode.

(2 marks)

c) An AlGaAs/GaAs *pin* photo-diode has an intrinsic GaAs layer of width 1 µm. The p$^+$ layer is very small at 10 µm. The diode is reverse-biased by a voltage of 1.75 V and then illuminated with short optical pulse of wavelength 700 nm.

- i) Estimate the absorption coefficient of the photo-diode.
- ii) Deduce the penetration depth to know if absorption will occur across the whole width.
- iii) Calculate the electric field across the intrinsic layer width
- iv) Estimate the hole drift velocity
- v) Hence, calculate the duration of the photo-current across the whole width of the diode.

(2+1+2+1+2 marks)

d) A photo-diode has an active light receiving area of 15 mm^2. When radiation of wavelength 650 nm and intensity 0.5 mW/cm^2 is incident on it, it generates a photo-current of 0.2 µA. What is the responsivity and quantum efficiency of the photo-diode at 650 nm?

(3+2 marks)

e) A 0.5 cm^2 photo-detector is irradiated with 750 nm light of intensity 75 mW/cm^2. Assuming that each photon generates one EHP, calculate the number of pairs created per second.

(3 marks)

f) What is the thickness of a Germanium crystal layer that is needed for absorbing 75% of the incident radiation at 1.5 µm? *(For Ge, $\alpha = 5.2 \times 10^5\ m^{-1}$ at 1.5 µm incident radiation)*

(2 marks)

******END OF EXAM PAPER******

UNIVERSITY of TECHNOLOGY, MAURITIUS

B.Eng (Hons.) Electronic Engineering

Cohort: BEE/13/FT

Examinations for 2016 – 2017 / Semester 2

MODULE: **OPTOELECTRONICS**
MODULE CODE: ELEC4105
Duration: 2½ Hours
Reading time: None

Instructions to Candidates:

1. Attempt ALL **FOUR** questions.
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3. All questions carry **equal** marks.
4. Maximum Marks = **100**
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6. Appendix is provided.

This question paper contains 4 questions and 5 pages.

ATTEMPT ALL FOUR QUESTIONS

QUESTION 1: (25 MARKS)

a) Describe **two** mechanisms by which electrons can be excited from the valence band to the conduction band?

(4 marks)

b) Given that the direct-band gap energy for Zinc Phosphide (Zn_3P_2) is **1.5 eV** at room temperature, calculate the phonon energy and momentum that you can expect from such a material with a lattice constant of **8.0889 Å** and velocity of sound of **330 m/s**.

(3+2 marks)

c) Given a **n-type** semiconductor with N_A and N_D atom concentrations of $3x10^9$ cm^{-3} and $7x10^{11}$ cm^{-3} respectively, calculate the hole concentration assuming $n_i = 1.4x10^{10}$ cm^{-3}.

(3 marks)

d) What do you understand by the Fermi Level of a crystal? According to the Fermi-Dirac function, what is the temperature at which there is **5%** probability that a state, with an energy **0.5 eV** above the Fermi energy, will be occupied by an electron?

(7 marks)

e) Given a piece of **p-type** silicon at **300 ºK**,
    i. Calculate the thermal velocity of a hole with an effective mass of $1.15m_0$ and a mobility of $0.045\ m^2V^{-1}s^{-1}$.
    ii. What is the electric field that needs to be applied to make a hole drift at a velocity of $90\ ms^{-1}$?

(4+2 marks)

QUESTION 2: (25 Marks)

a) Describe **three** advantages of LED lighting over conventional incandescent lighting.

(3 marks)

b) State **two** impairments that reduces the radiant flux of a surface-emitting LED. Illustrate one way how any one of these impairments may be addressed?

(4 marks)

c) Illustrate how white light can be generated using a Blue Gallium Nitride *(GaN)* LED. Give **two** advantages of this technique.

(3+2 marks)

d) The bangap energy Eg in eV for $In_{1-x}Ga_xAs_yP_{1-y}$ lattice-matched to ***InP*** is given by the empirical relationship $E_g = 1.35 - 0.72y + 0.12y^2$ where $y=2.15(1-x)$. Calculate the fraction of **Gallium** suitable for a **1.15 µm** emitter.

(6 marks)

e) Given a particular Indium Gallium Aluminium Phosphide *(InGaAlP)* edge-emitting LED drawing a current **50 mA** when the voltage is **1.5 V** and operating at **560 nm** wavelength with optical power that is coupled in a multi-mode fibre through a lens is **1.5 mW**.

i) What is the colour of the light emitted from the LED?
ii) Calculate the overall efficiency of the LED.
iii) Calculate its internal quantum efficiency given that $T_r = 15\ ns$ and $T_{nr} = 40\ ns$.
iv) Hence, calculate the internal optical power generated by the LED.

(1+2+1+3 marks)

QUESTION 3: (25 Marks)

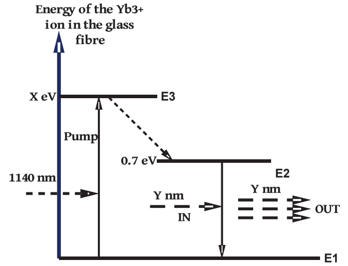
a) State **four** general properties of lasers.

(4 marks)

b) Describe why '*population inversion*' is a necessary condition for sustained lasing action?

(4 marks)

c) Consider a hypothetical Ytterbium Doped optical Fibre Amplifier (YDFA) below:



i) Calculate the value of X in eV at level E3.
ii) Which level is the meta-stable state?
iii) Explain what happens between level E3 and E2 as shown by the slanted dotted arrow.
iv) Calculate the value of Y in nm between level E2 and E1?

(2+1+3+2 marks)

d) A particular 50 W Krypton-Fluoride laser operating at 248 nm has a tube that is 50 cm long. The operating temperature is at 75 °C. It operates at a voltage of 2 kV and carries a current of 500 mA. (Mass of an Krypton atom = 1.39×10^{-25} kg)

i) What is the efficiency of the laser?
ii) Estimate the Doppler broadened linewidth (Δv) in the output spectrum.
iii) What are the mode number m values that satisfy the resonant cavity condition?
iv) What are the mode separations in frequency (Δv_m) and wavelength ($\Delta\lambda_m$)?

(1+3+1+4 marks)

QUESTION 4: (25 Marks)

a) State **five** desirable requirements of a generic photo-detector.

(5 marks)

b) A silicon *pin* photo-diode has an intrinsic layer of width 20 μm. The p^+ layer is very small at 0.5 μm. The diode is reverse-biased by a voltage of 100 V and then illuminated with short optical pulse of wavelength 900 nm.

   i) Estimate the absorption coefficient of the photo-diode.
   ii) Deduce the penetration depth to know if absorption will occur across the whole width.
   iii) Calculate the electric field across the intrinsic layer width
   iv) Estimate the hole drift velocity
   v) Hence, calculate the duration of the photo-current across the whole width of the diode.

(2+2+2+2+2 marks)

c) A photo-diode has an active light receiving area with diameter 1 mm. When radiation of wavelength 900 nm and intensity 0.3 mW/cm^2 is incident it generates a photo-current of 0.25 μA. What is the responsivity and quantum efficiency of the photo-diode at 900 nm?

(3+2 marks)

d) A 0.5 cm^2 photo-detector is irradiated with 840 nm light of intensity 40 mW/cm^2. Assuming that each photon generates one EHP, calculate the number of pairs created per second.

(3 marks)

e) What is the thickness of a Germanium crystal layer that is needed for absorbing 90% of the incident radiation at 1.5 μm? ***(For Ge, $\alpha = 5.2\ x10^5\ m^{-1}$ at 1.5 μm incident radiation)***

(2 marks)

******END OF EXAM PAPER******

Constants:

$h = 6.626\ x10^{-34}\ m^2kg/s$
$V_T = kT/q = 25\ \text{mV}\ @\ 300K$
$q = 1.6x10^{-19}\ C$
$k = 1.38x10^{-23}\ J/K$
$m_0 = 9.1x10^{-31}\ kg$
$c = 3x10^8\ m/s$
$N_A = 6.02x10^{23}\ mol^{-1}$

Miscellaneous Formulae:

$$n = p = n_i$$

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

$$p_p = \frac{1}{2}[N_A - N_D + \sqrt{(N_A - N_D)^2 + 4n_i^2}]$$

$$I_d = nqV_dA$$

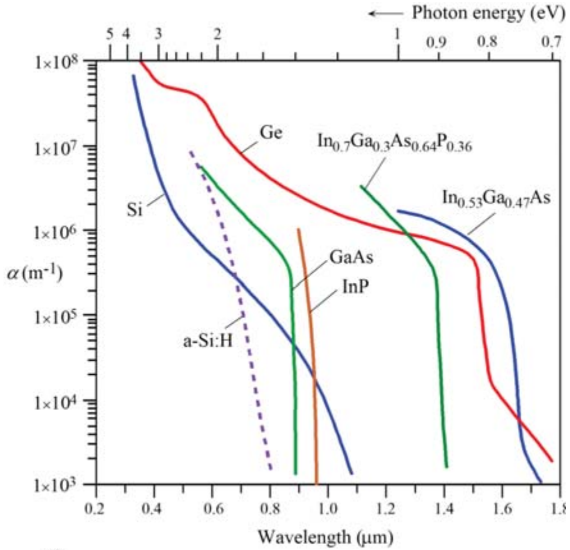
$$V_{th} = \sqrt{\frac{3kT}{m^*}}$$

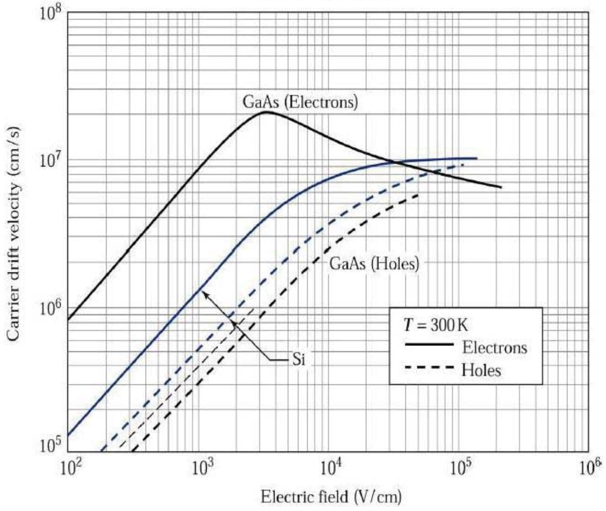
$$\delta V = 2v_0\sqrt{\frac{2kT\ln 2}{Mc^2}}$$

$$E_g = \frac{hc}{\lambda} - k_BT$$

$$\eta_{int} = \frac{\text{Rhc}}{\text{e}\lambda}$$

$$R = \frac{I_{ph}}{P_0}$$


← Photon energy (eV)
5
4
3
2
1
0.9
0.8
0.7
1×10⁸
1×10⁷
1×10⁶
1×10⁵
1×10⁴
1×10³
α (m⁻¹)
Ge
In0.7Ga0.3As0.64P0.36
In0.53Ga0.47As
Si
GaAs
InP
a-Si:H
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
Wavelength (µm)


10⁸
10⁷
10⁶
10⁵
Carrier drift velocity (cm/s)
GaAs (Electrons)
GaAs (Holes)
Si
T = 300 K
Electrons
Holes
10²
10³
10⁴
10⁵
10⁶
Electric field (V/cm)