CAN 1011: Data Communication

- Bandwidth, Channel Capacity
- Transmission Impairments

Contents

- Bandwidth and data rate
- The decibels notation for signal strength
- Transmission Impairments
- Channel Capacity

Maximum Data Rate (Channel capacity)

Considerations

- Bandwidth of transmission system
- Signal to noise ratio (SNR)
- Receiver type
- Specified acceptable error performance

Bandwidth & Data Rates: Trade-offs…

- Increasing the data rate (bps) while keeping BW the same (to economize) means working with inferior (poorer) waveforms at the receiver, which may require:
  - Ensuring higher signal to noise ratio (SNR) at RX
    - Larger transmitted power (may cause interference to others!)
    - Shorter link distances
    - Use of more en-route repeaters/amplifiers
    - Better shielding of cables to reduce noise, etc.
  - Using a more sensitive (& costly!) receiver
  - Suffering from higher bit error rates
    - Tolerate them?
    - Add more efficient means for error detection and correction- **this also increases overhead!.**

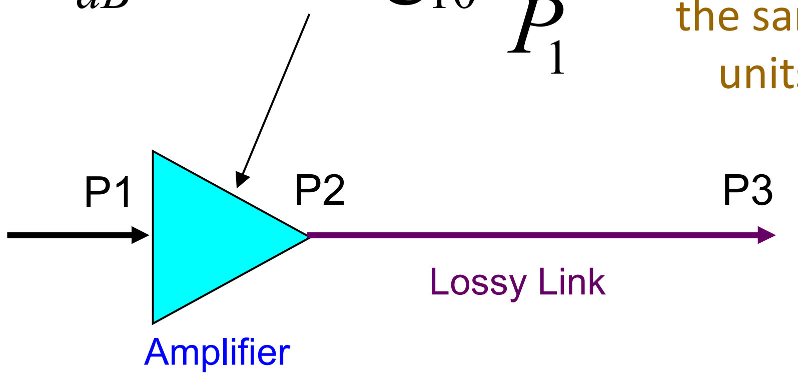
Decibels & Signal Strength

- The decibel notation (dB) is a logarithmic measure of the ratio between two signal power levels
  - N_{dB} = number of decibels
  - P_1 = input power level (Watts)
  - P_2 = output power level (Watts)

$$N_{dB} = 10\log_{10}\frac{P_2}{P_1}$$

P1, P2 Should have the same units

- e.g. → Amplifier gain
  → Signal loss (attenuation) over a link



Example:

- A signal with power level of 10mW is inserted into a transmission line
- Measured power some distance away is 5 mW
- Power "gain" in dBs is expressed as 10 log (out/in)

N_{dB}=10 log (5/10)=10(-0.3)= -3 dB (negative gain is loss)

- negative dBs: $P_3 < P_2$ (Loss), positive dBs: $P_2 > P_1$ (Gain)

Relationship between dB Values and Power ratio (P_2/P_1)

Log (Compressed)

	Power Ratio	dB	Power Ratio	dB
1	1	0		
	10^1	10	10^{-1}	-10
	10^2	20	10^{-2}	-20
	10^3	30	10^{-3}	-30
	10^4	40	10^{-4}	-40
	10^5	50	10^{-5}	-50
1,000,000	10^6	60	10^{-6}	-60
	2	3	1/2	-3

Linear

Decibels and Signal Strength

- Decibel notation is a relative (not absolute) measure:
  - A loss of 3 dB halves the power (e.g. 100 to 50,   16 to 8, …)
  - A gain of 3 dB doubles the power (e.g. 5 to 10,   7.5 to 15, …)
- Will see shortly how we can handle absolute levels
- Advantages of using dBs:
  - The "log" allows replacing:
    - Multiplication with Addition

      C = A * B

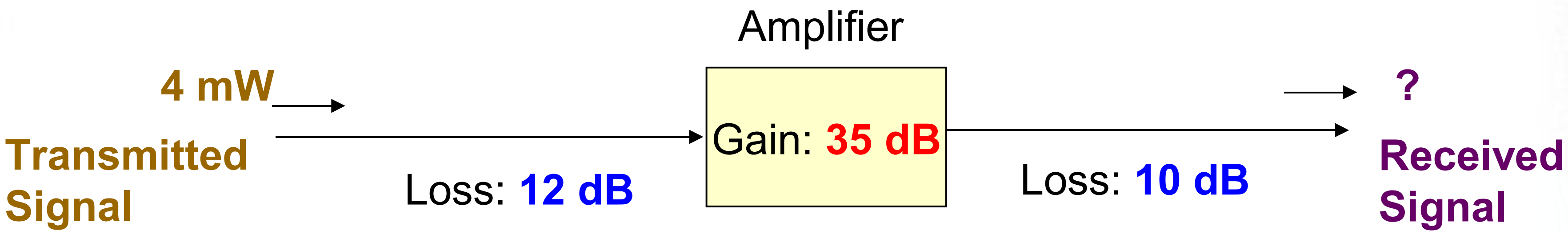
      Log C = Log A + Log B
    - and Division with Subtraction

      A = C / B

      Log A = Log C - Log B

Decibels and Signal Strength

- Example: Transmission line with an intermediate amplifier



- Net power gain over transmission path:

  + 35 – 12 – 10 = 13 dB (positive means there is net gain)

$$\therefore \quad 13 = 10\log_{10}\left[\frac{\text{Received Signal Power}}{4mW}\right]$$

$$\left[\frac{\text{Received Signal Power}}{4mW}\right] = \log^{-1}{}_{10}\left[\frac{13}{10}\right]$$

- Received signal power = (4 mW) $\log_{10}^{-1}(1.3)$ = 4 x $10^{1.3}$

  = 4 x $10^{1.3}$ mW = 79.8 mW

Transmission Impairments

- Signal received is often a degraded form of the signal transmitted
- Why? What happens en-route?... Impairments:
  - Attenuation:
    - Limits the bandwidth of the received signal
    - In-band signals arrive weaker
    - Attenuation distortion (Attenuation is not uniform over bandwidth)
  - Delay
  - Delay distortion
  - Noise and interference (including crosstalk)
- Effect:
  - On analogue data - Some degradation in signal quality
  - On digital data – Fatal bit errors (total bit reversals)

Attenuation

- Signal strength falls off with distance traveled
- Nature of loss in signal power depends on medium:
  - Guided (Wires, etc.):

    Signal power after travelling distance d

    - Exponential drop is signal power with distance: $P_d = P_0 e^{-\alpha d}$

      $10 \ln (P_d/P_0) = -\alpha d$

      $10 \log (P_d/P_0) = -\alpha' d$

    $\rightarrow$ Loss: α' dBs per km (α' depends on medium type e.g. fiber, twisted pair, cable)
  - Unguided (Open-space):
    - Inverse square law spread with distance: $P \alpha P_0 /d^2$

    $\rightarrow$ Loss: 6 dBs for each distance doubling
    - Absorption and scattering by objects
    - May also depend on weather, e.g. rain, sunspots, etc

Effects of Attenuation

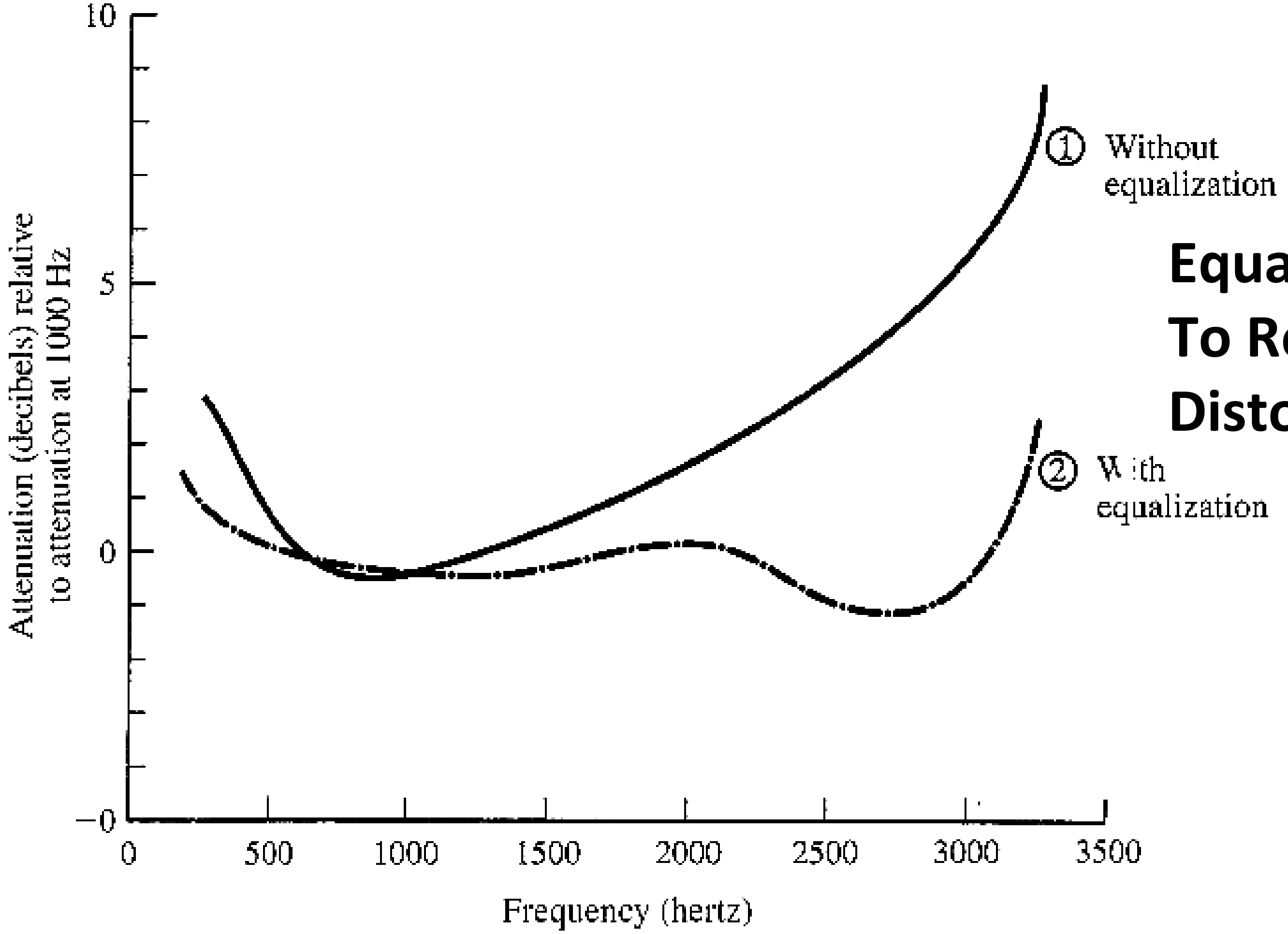
- Received signal strength must be:
  - Large enough to be detected
  - Sufficiently higher than noise to be interpreted correctly (without error)
- To overcome these problems:
  - Use amplifiers (analogue transmission mode)

    or repeaters (digital transmission mode) en-route
  - Amplifier gains should not be too large as this may cause signal distortion due to saturation (non-linearities)
  - Problem with networks: distance actually travelled (hence attenuation) will depend on actual route taken through the network!

Attenuation Distortion

- Attenuation usually increases with frequency
- This causes bandwidth limitation (understood)
- Moreover, over the transmitted bandwidth itself:
  - Different frequency components of the signal get attenuated differently → Signal distortion
  - Affects analogue signals more
- To overcome this problem:
  - Use **equalizers** that reverse the effect of frequency-dependent attenuation distortion:
    - **Passive**: e.g. loading coils in telephone circuits
    - **Active**: Amplifier gain designed specifically for this purpose

Attenuation Distortion

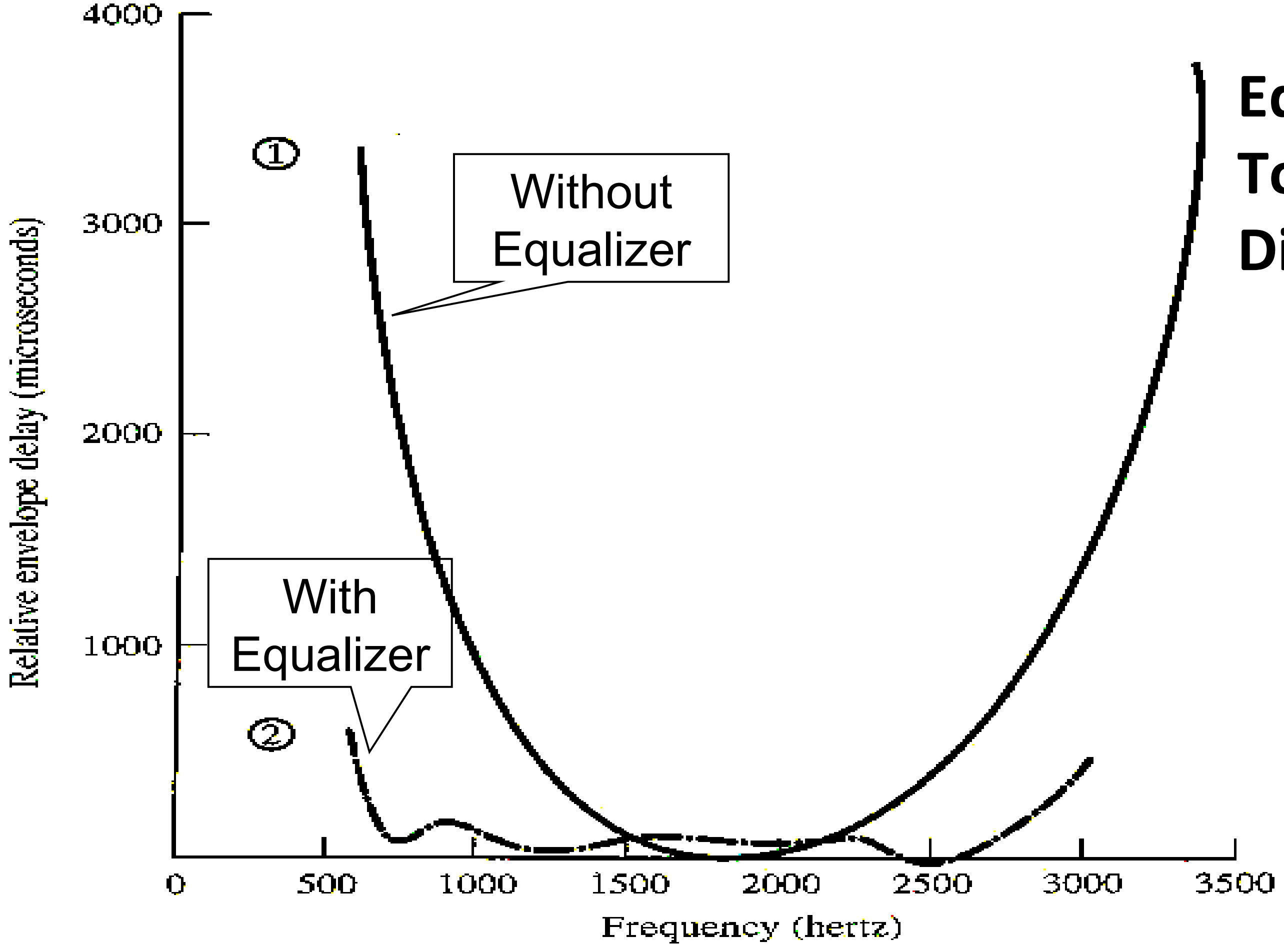


Equalization To Reduce Attenuation Distortion

Delay Distortion

- Happens only on guided media
- Wave propagation velocity varies with frequency:
  - Highest at the centre frequency (minimum delay)
  - Lower at both ends of the bandwidth (larger delay)
- Effect: Different frequency components of the signal arrive at slightly different times! (Dispersion in time)
- Affects digital data more: due to bit spill-over (timing is more critical here than for analogue data)
- Again, equalization can help overcome the problem

Delay Distortion



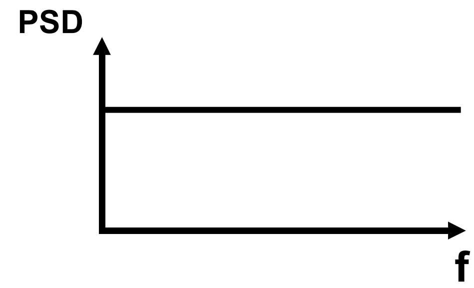
Equalization To Reduce Delay Distortion

Noise

- Definition: Any additional unwanted signal inserted between transmitter and receiver
- The most limiting factor in communication systems
- Noise Types:
  - Thermal Noise
  - Inter-modulation Noise
  - Crosstalk Noise
  - Impulse Noise

Noise

- Thermal (White) Noise
  - Due to thermal agitation of electrons (Increases with temperature)
  - **Uniformly distributed over frequency** (White noise)
    $\rightarrow$ Difficult to eliminate
    (exists even in the same bandwidth as your signal!, gets amplified!)
  - Effect is more significant on weak received signals, e.g. from satellites

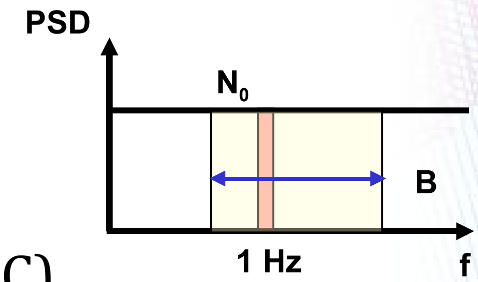


Thermal Noise, Contd.

- Thermal noise power density in 1 Hz of bandwidth, N_0 (Constant, Independent of frequency):

  $$N_0 = kT \quad (W/Hz)$$

  - k Boltzmann's constant = 1.38×10^{-23} J/K
  - T temperature in degrees Kelvin (= 273 + t °C)



- Thermal noise power in a bandwidth of *B* Hz:

$$N = N_0 B = kTB \quad \text{(watts)}$$

$$= -228.6 + 10\log T + 10\log B \quad \text{(dBw)}$$

10 log k

Can you see some disadvantage now in having a larger BW?

Example: at t = 21 °C (T = 294 °K) and for a bandwidth of 10 MHz:

N = -228.6 + 10 log 294 + 10 log 10^7

= - 133.9 dBW

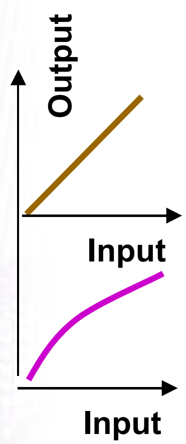
Noise

– Inter-modulation Noise

- Signals having the sum and difference (frequency mixing) of original frequencies sharing a transmission system

  $f1, f2 \rightarrow (f1+f2)$ and $(f1-f2)$

- Caused by non-linearities in the medium and equipment, e.g. due to overdrive and saturation of amplifiers
- Danger: Resulting new frequency components may fall within valid signal bands, thus causing interference



$A \cos \theta 1 + B \cos \theta 2$ → **Linear System** → $A' \cos \theta 1 + B' \cos \theta 2$

$K(A \cos \theta 1 + B \cos \theta 2)$

$A \cos \theta 1 + B \cos \theta 2$ → **Non-Linear System** → $A' \cos \theta 1 + B' \cos \theta 2 + f(2\theta 1)+f(2\theta 2)+f(\theta 1-\theta 2)+f(\theta 1+\theta 2)$

$K(A \cos \theta 1 + B \cos \theta 2) + K(A \cos \theta 1 + B \cos \theta 2)^2$

Inter-modulation components

New spurious components can fall within genuine signal bands causing interference

Noise

- Crosstalk Noise
  - A signal from one channel picked up by another channel in close *proximity*
  - Examples:
    - Physical proximity: coupling between adjacent twisted pair channels

    → Shield cables properly
    - Directional proximity: antenna pick up from other directions

      → Use directional antennas
    - Spectral proximity: leakage between adjacent channels in frequency division multiplexing (FDM) systems

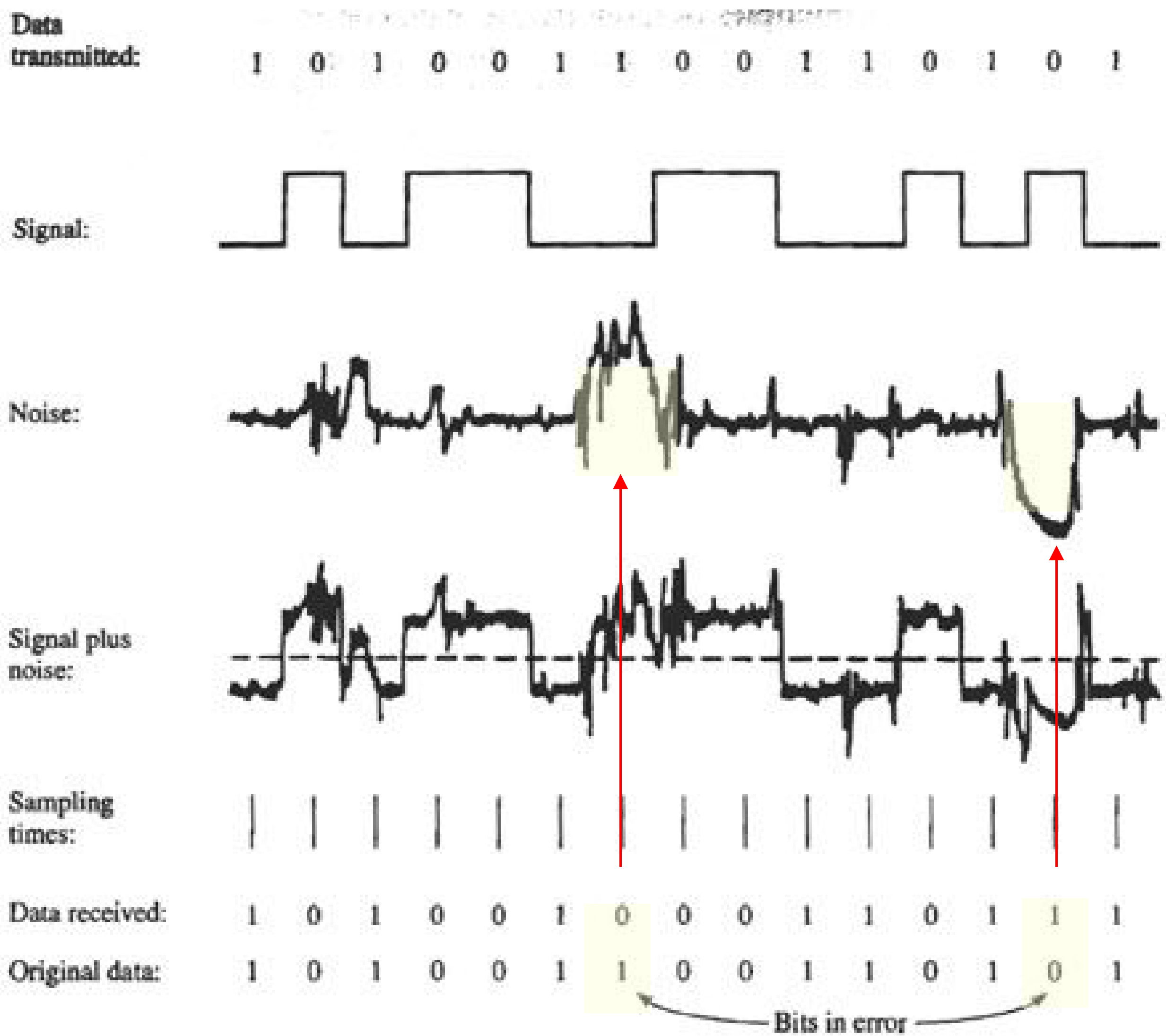
    → Use guard bands between adjacent channels

Noise

- Impulse Noise
  - Pulses (spikes) of irregular shape and high amplitude lasting short durations
  - Causes: External electromagnetic interference due to switching large currents, car ignition, lightning, …
  - Minor effect on analog signals (e.g. crackling noise in voice channels)
  - Major effect on digital signals- Bit reversal error!
  - More damage at higher data rates

  (a noise pulse of a given width can destroy a larger block of bits)

Effect of Impulse Noise on a Digital Signal



Q: What is the effect of the same noise at 10 times the data rate?

Channel Capacity

- Channel capacity: Maximum data rate usable under a given set of communication conditions
- How channel BW (B), signal level, noise and impairments, and the amount of data error that can be tolerated limit the channel capacity?
- In general, maximum possible data rate, C, on a given channel

  = Function (B, Signal wrt noise, Bit error rate allowed)
  - Maximum data rate: Maximum rate at which data can be communicated on the channel, bits per second (bps)
    - Bandwidth: BW of the transmitted signal as constrained by the transmission system, cycles per second (Hz)
    - Signal relative to Noise, SNR = signal power/noise power ratio (<u>Higher</u> SNR → <u>better</u> communication conditions → higher C)
    - Bit error rate (BER) <u>allowed</u>: in (bits received in error)/(total bits transmitted). Equal to the bit error probability.
      e.g. Higher allowed → higher usable data rates → higher C

Channel Capacity, C

- So, in general: C bps = F(B, SNR, BER)
- Three formulations under different assumptions:

Idealistic

Assumptions	Formulation
Ideal: Noise-free, Error-free: C = F(B)	Nyquist
Noisy: Error-free: C = F(B, SNR)	Shannon
Practical: Noisy, Error: C = F(B, SNR, BER)	E_b/N_0 vs Error Rate

Realistic

Bandwidth or Spectral Efficiency (BE)

$$BE = \frac{Channel\ Capacity\ C}{Bandwidth\ B}, \qquad bps / Hz$$

- Measures how well we are utilizing a given bandwidth to send data at a high rate….
- Can be greater than 1 (not like engineering efficiencies)
- The larger the better

Nyquist Capacity (Noise-free, Error-free)

- Idealized, theoretical, assumes a noise-free → error-free channel
- Nyquist showed that (without noise, without errors): If rate of signal transmission is 2B then a signal with frequency components up to B Hz is sufficient to carry that signalling rate
- In other words: Given bandwidth B, highest signalling rate possible is 2B signal elements per second
- How much data rate does this represent?

(depends on how many bits are represented by each signal element!)

  - Given a binary signal (1,0), data rate is same as signal rate → Data rate supported by a BW of B Hz is 2B bps → C = 2B
  - For the same B, data rate can be increased by sending one of M different signals (symbols): as each signal level now represents $\log_2 M$ bits

- Generalized Nyquist Channel Capacity, $C = 2B \log_2 M$ bits/s (bps)
- Bandwidth efficiency = $C/B = 2 \log_2 M$, dimensionless quantity

Nyquist Bandwidth: Example

- $C = 2B \log_2 M$ bits/s
  - C = Nyquist Channel Capacity
  - B = Bandwidth
  - M = Number of discrete signal levels (symbols) used
- Data on telephone channel:
- B = 3400-300 = 3100 Hz
- With a binary signal (M = 2 levels)

  $C = 2B \log_2 2 = 2B \times 1 = 6200$ bps
- With a quadnary signal (M = 4 levels)

  $C = 2B \log_2 4 = 2B \times 2 = 4B = 12{,}400$ bps
- Channel capacity increased, but larger number of signal levels (M) makes it more difficult for the receiver to determine data correctly in the presence of noise

Shannon Capacity (Noisy, Error-Free)

- Highest error-free data rate in the presence of noise
- Signal power to noise power S/N = signal/noise levels

$$SNR_{dB} = 10 \log_{10} (S/N)$$

- Errors are less likely with lower noise (larger SNR). This allows higher error-free data rates i.e. larger Shannon channel capacities
- Shannon Capacity $C = B \log_2(1+S/N)$

Caution! Log_2 Not Log_{10}

- For a given BW, the larger the SNR the higher the data rate I can use without introducing errors
- C/B: Spectral (bandwidth) efficiency, BE, (bps/Hz) (>1)
- Larger BEs mean better utilization of a given bandwidth B for transmitting data fast.

Shannon Capacity: Comments

- Formula says: for data rates ≤ calculated C, it is theoretically possible to find an encoding scheme that achieves error-free transmission at the given SNR… But it does not say how!

  It is a theoretical approach based on thermal (white) noise only. But in practice, we also have impulse noise, attenuation and delay distortions, etc…
  - So, maximum error-free data rates measured in practice are expected to be lower than the C predicted by the Shannon formula due to the greater noise
- However, maximum error-free data rates can be used to compare practical systems: The higher that rate the better the system…

Shannon Capacity: Comments

- Formula suggests that changes in B and SNR can be done arbitrarily and independently… but

→ In practice, this may not be the case!

  - Higher SNR obtained through excessive amplification may also introduce non-linearities → increased distortion and inter-modulation noise … which reduces SNR!
  - High Bandwidth B opens the system up for more thermal noise (kTB), and therefore reduces SNR!

Shannon Capacity: Example

Spectrum of communication channel extends from 3 MHz to 4 MHz

SNR = 24dB

Then B = 4MHz – 3MHz = 1MHz, SNR_{dB} = 24dB = 10 $\log_{10}$ (S/N)

S/N = $\log^{-1}_{10}$ (24/10) = $10^{24/10}$ = 251

- Using Shannon's formula: C = B $\log_2$ (1+ S/N)

  C = 10^6 * $\log_2$(1+251) ~ 10^6 * 8 = 8 Mbps

- Based on Nyquist's formula, determine M that gives the above channel capacity: C = 2B $\log_2$ M

  8 * 10^6 = 2 * (10^6) * $\log_2$ M

  4 = $\log_2$ M, therefore M = 16

E_b/N_0 vs Error Rate Formula

- Handling both noise and a quantified error rate simultaneously
- We introduce E_b/N_0: A standard quality measure of three channel parameters (B, SNR, R) and can also be independently related to the error rate

  R is the data rate. Max value of R is the channel capacity C
- It expresses SNR in a manner related to the data rate, R
  - E_b = Signal energy in one bit interval (Joules)

    = Signal power (Watts) x bit interval T_b (second) $T_b = 1/R$

    = S x (1/R) = S/R
  - N_0 = Noise power (watts) in 1 Hz = kT.

$$\frac{E_b}{N_0} = \frac{ST_b}{N_0} = \frac{S/R}{kT} = \frac{S}{kTR} \qquad \frac{E_b}{N_0} = \frac{S/R}{N_0} = \frac{S}{N}\frac{B_T}{R} = SNR\left(\frac{B_T}{R}\right)$$

= SNR/BE

E_b/N_0

- Bit error rate for digital data is a decreasing function of E_b/N_0 for a given signal encoding scheme

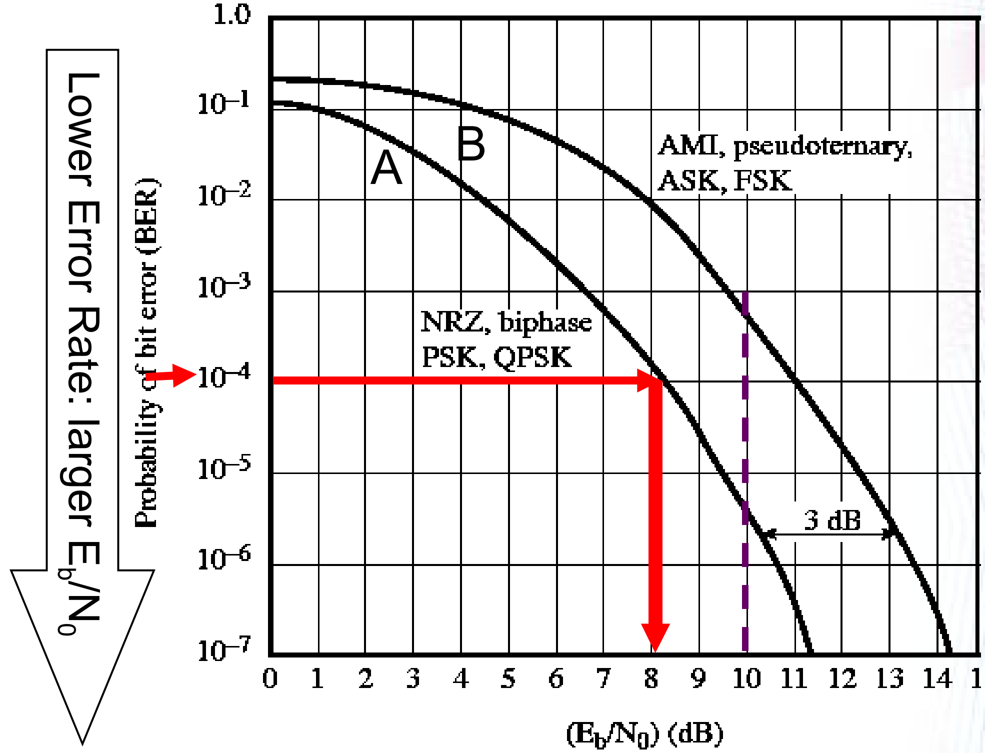
- **Analysis**: ⇒ For a given system (SNR, B, R) → (E_b/N_0), determine error rate BER

- **Design**: ⇒ Given a desired error rate BER, get E_b/N_0 to achieve it, then determine other parameters from formula, e.g. S, SNR, R, etc.

- Effect of S, R, T on error performance

- Which encoding scheme is better: A or I?

BER vs E_b/N_0 curve for a given encoding scheme



$$\left(\frac{E_b}{N_0}\right)_{dB} = S_{dBW} - 10\log R - 10\log k - 10\log T$$

$$= S_{dBW} - 10\log R + 228.6\ dBW - 10\log T$$

$$\frac{E_b}{N_0} = \frac{S/R}{N_0} = \frac{S}{N}\frac{B_T}{R} = SNR\left(\frac{B_T}{R}\right) = \frac{\text{SNR}}{\text{BE}}$$

Max R = C, BE = C/B

Example:

$$\left(\frac{E_b}{N_0}\right)_{dB} = S_{dBW} - 10\log R - 10\log k - 10\log T$$

$$= S_{dBW} - 10\log R + 228.6\ dBW - 10\log T$$

- Given:
  - The effective noise temperature, T, is 290°K
  - The data rate, R, is 2400 bps
  - Would like to operate with a bit error rate of 10^{-4} (e.g. 1 error in 10^{4} bits)

What is the minimum signal level required for the received signal?

- From curve, a minimum E_b/N_o needed to achieve a bit error rate of 10^{-4} = 8.4 dB

- 8.4 = S(dBW) – 10 log 2400 + 228.6 dBW – 10 log290
  = S(dBW) – (10)(3.38) + 228.6 – (10)(2.46)

S = -161.8 dBW

E_b/N_0 in terms of BE, assuming Shannon channel capacity

- From Shannon's formula:

$$C = B \log_2(1+SNR)$$

We have:

$$SNR = (2^{C/B} - 1) = (2^{BE} - 1)$$

- From the E_b/N_0 formula:

$$\frac{E_b}{N_0} = \frac{SNR}{BE} = \frac{1}{BE}(2^{BE} - 1)$$

C/B (bps/Hz) is the spectral (bandwidth) efficiency BE based on Shannon channel capacity

Example

- Find the minimum E_b/N_0 required to achieve a Shannon bandwidth efficiency (BE=$C_{Shannon}$/B)of 6 bps/Hz:

$$\frac{E_b}{N_0} = \frac{1}{BE}(2^{BE} - 1)$$

- Substituting in the equation above:

$E_b/N_0 = (1/6)\ (2^6 - 1) = 10.5 = 10.21$ dB