Wireless Communications

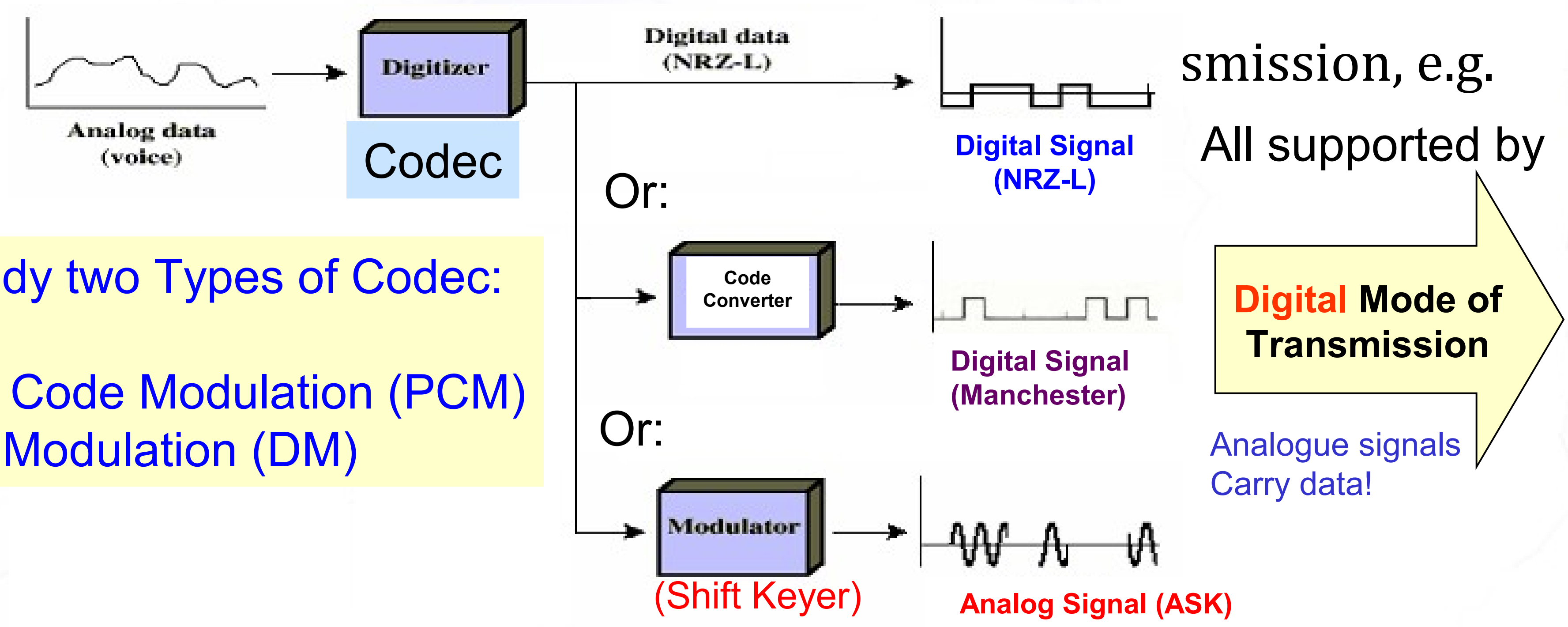
- Encoding & Modulation (Part 2)

Contents

- Encoding and Modulation (Part II)
  - Analogue data into Digital signals
  - Analogue data into Analogue signals

Analogue Data, Digital Signal

- Digitization
  - Conversion of analogue data into signals suitable for the digital mode of transmission/storage
    - The digital data can be transmitted digitally as is (e.g. NRZ-L)
    - Or converted to a more appropriate digital code, e.g. Manchester



smission, e.g.

All supported by

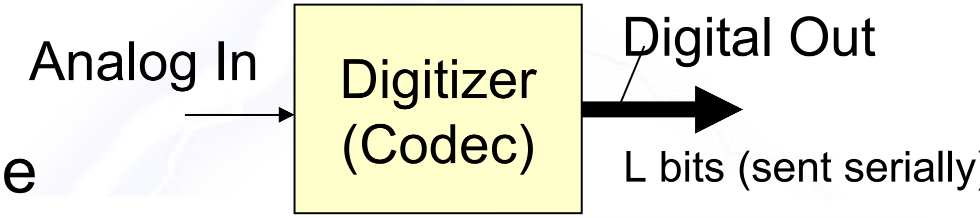
Will study two Types of Codec:

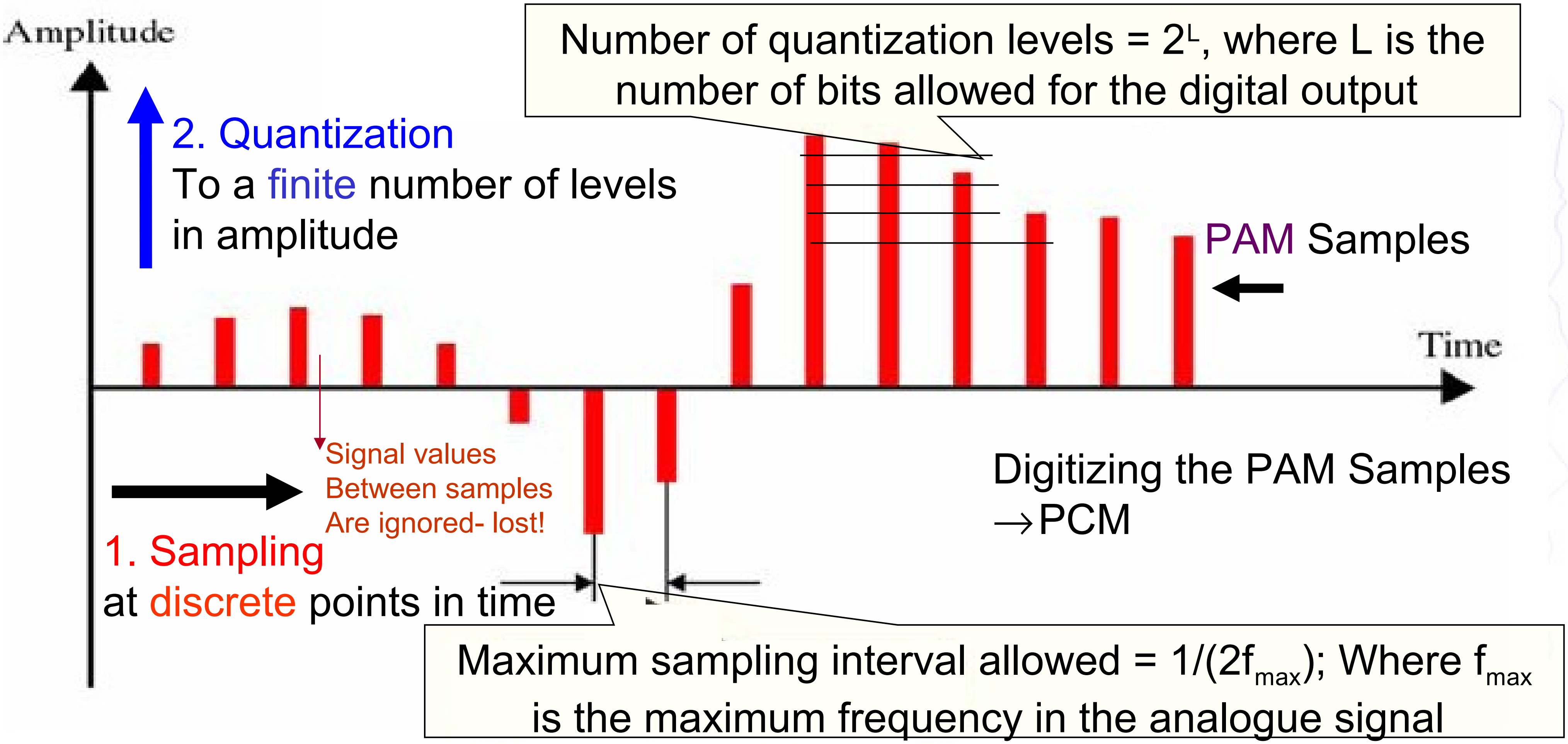
- Pulse Code Modulation (PCM)
- Delta Modulation (DM)

Two basic tasks to be performed by a digitizer:

"Analogue" is continuous in **both** time and amplitude… Must discretize it in both!

- 1. Sampling in time
- 2. Quantization in amplitude





$2f_{max}$ is the minimum acceptable sampling rate

Sampling

- **Nyquist Sampling Theorem:**

  If a signal is sampled at regular intervals at a rate higher than *twice* the **highest** signal frequency f_{max}, the samples contain all the information in the original signal
- Original signal may be reconstructed from these samples using an **ideal** low-pass filter
- Example: Voice data limited to 4000Hz
  - Requires sampling at a rate of at least

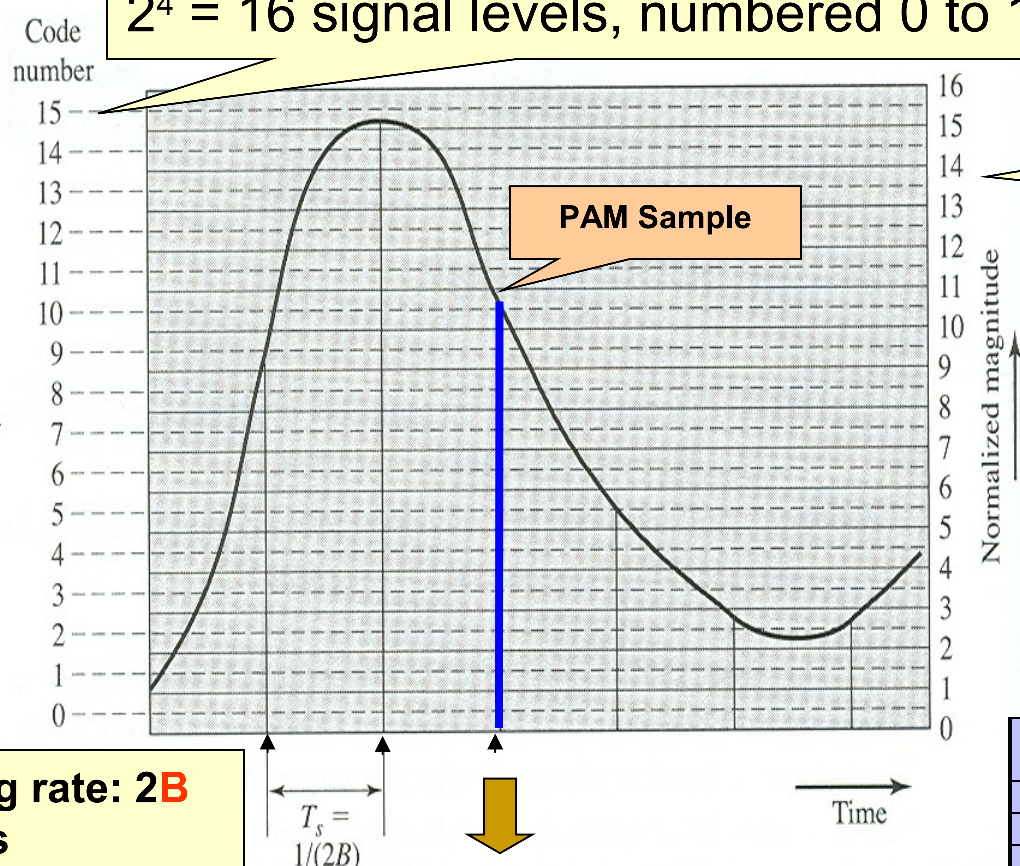
    8000 sample per second

Quantization using 4 bits

2^4 = 16 signal levels, numbered 0 to 15

Quantization

Level numbers starting from 0



Signal Amplitude, Volts
Vmax = 16 V

Sampling rate: 2B sample/s

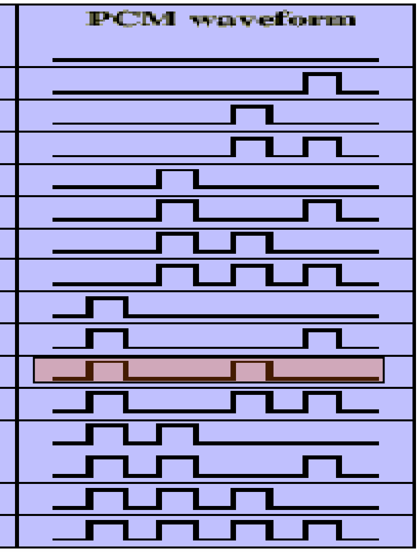
PAM value	1.1	9.2	15.2	10.8	5.6	2.8	2.7
Quantized code number	1	9	15	10	5	2	2
PCM code	0001	1001	1111	1010	0101	0010	0010

Data Rate: 2B x 4 bps

Figure 5.16 Pulse Code Modulation Example

Transmitted **Serial Code** representing the value of the PAM Samples:

Digit	Binary Equivalent	PCM waveform
0	0000	
1	0001	
2	0010	
3	0011	
4	0100	
5	0101	
6	0110	
7	0111	
8	1000	
9	1001	
10	1010	
11	1011	
12	1100	
13	1101	
14	1110	
15	1111	



Each PAM sample is assigned the number of the nearest quantization level and the corres. digital code is transmitted

Must finish sending the n bits of the code within the sampling interval ….before the next sample starts!

Pulse Code Modulation (PCM)

- Start with the analogue sampled pulses (Pulse Amplitude Modulation, PAM)
- Assign each sample a digital value (= number of the closest quantization level)
- n = 4 bit system gives M = 16 levels ($M = 2^n$)
- Quantization error or noise
  - Larger for small M (number of levels)
  - Approximations mean it is impossible to recover the original signal exactly
  - SNR for quantization error using n bits is

$$SNR = 20\log_{10} 2^n + 1.76\,dB = 6.02\,n + 1.76\,dB$$

  - Each additional bit used for quantization increases SNR by about 6 dB (a power factor 4)
- 256 quantization levels: n = 8 bits, SNR ≈ 50 dB

→ Quality comparable with analogue transmission

- Voice: 2 x 4000 = 8000 samples per second, with of 8 bits per sample, this is a data rate of 8000*8 = 64 kbps

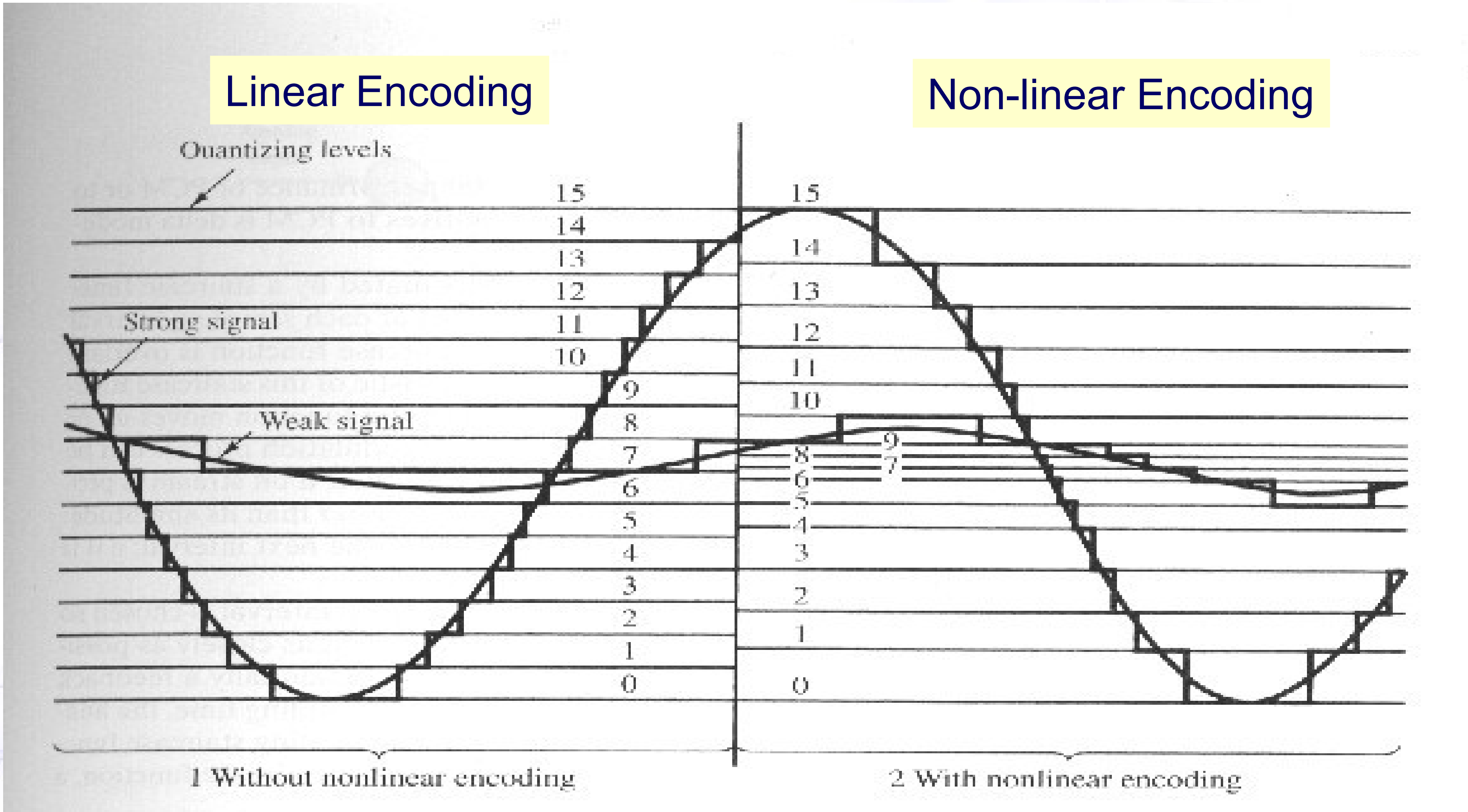
PCM Example

- Suppose we want to encode an analogue signal that has voltage levels 0 – 5 V using 2-bit PCM (n = 2 bits) (M = 2^2 = 4 levels)
- We divide the maximum voltage level into four intervals, so the size of each interval is 5/4=1.25 V
  - Level intervals: 0-1.25, 1.25-2.5, 2.5-3.75, 3.75-5
- We select the quantization levels at the middle of each level interval
  - i.e. selected levels are: 0.625, 1.875, 3.125, 4.375
  - This guarantees a maximum quantization error of ½ (5V /4) = 0.625 (=1/2 LSB)
  - and quantization SNR = 6 x 2 + 1.76 = 13.76 dB

Problem with Linear (Uniform) Encoding

- Absolute quantization error for each sample is the same regardless of signal level
  - Signals with lower amplitudes are relatively more distorted
- One Solution: make quantization levels not evenly spaced (denser for low amplitudes)
- i.e. higher number of quantization steps for lower amplitudes and smaller number for larger ones
- Reduces overall signal distortion
- This is non-linear encoding

Effect of Nonlinear Coding



Quantization error is fixed-
same for both weaker and stronger signals

Weaker signals have <u>smaller</u> quantization errors

Example

- Consider an audio signal with spectral components in the range of 300 to 3000 Hz. Assuming a sampling rate of 7000 samples per second will be used to generate the PCM signal
  - 7000 > 2 x 3000 → OK
  - To obtain a quantization SNR of 30 dB, what is the number of uniform quantization levels needed?
    - $(SNR)_{dB} = 6.02\,n + 1.76 = 30$ dB

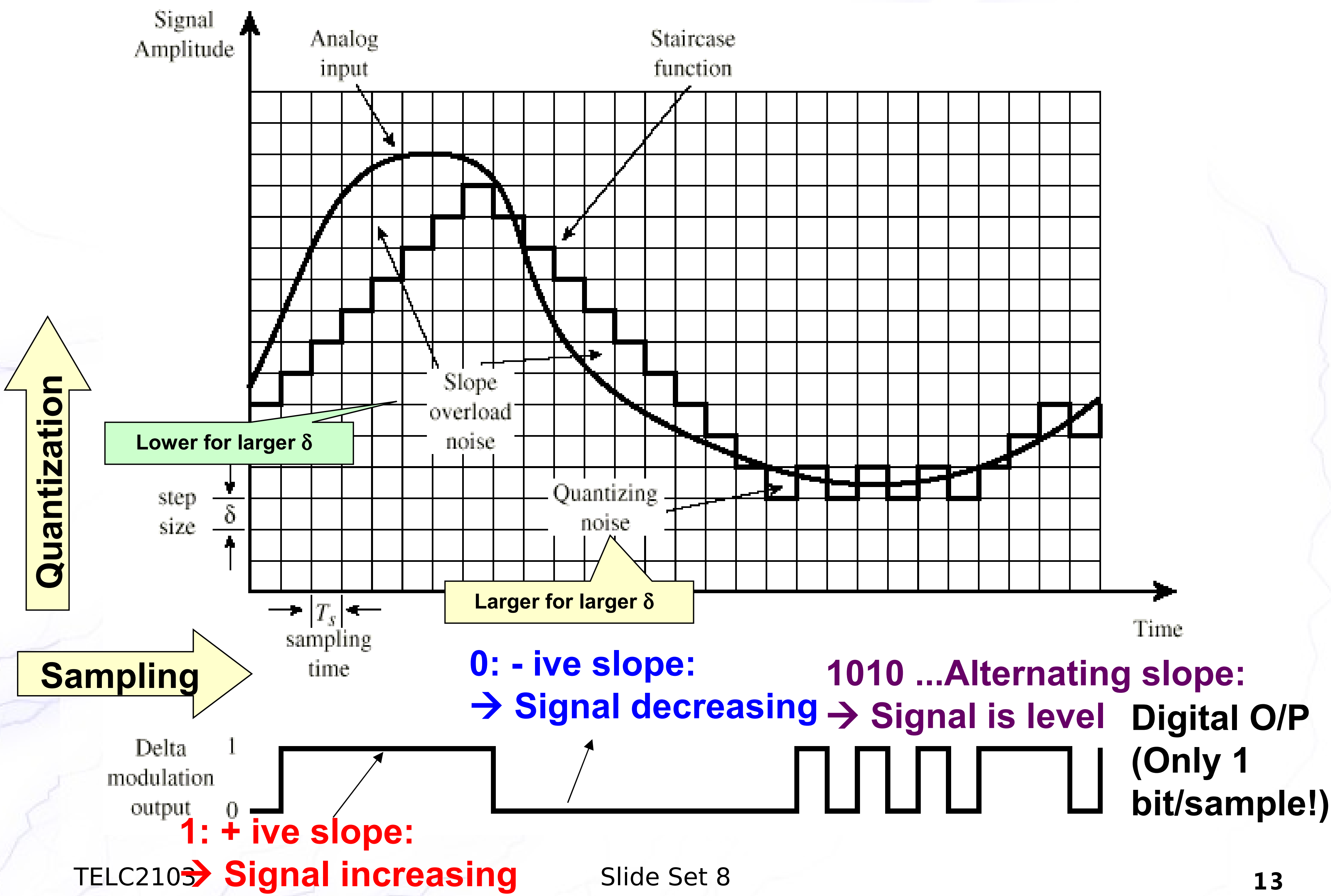
    $n = (30 - 1.76)/6.02 = 4.69$

    Always round off to the next higher integer $\Rightarrow n = 5$ bits $\Rightarrow 2^5 = 32$ quantization levels
  - What is the **data rate** required?
    - R = 7000 samples/sec × 5 bits/sample = 35 Kbps

Delta Modulation: An alternative to PCM

- An attempt to reduce complexity (and large R) for PCM
- Analog input is approximated by a staircase function
  - Move up or down one fixed amplitude increment (δ) at each sample interval to track changes in the analogue waveform
- A single bit stream is produced to approximate the derivative of the analogue signal rather than its amplitude
  - Generate a 1 if staircase goes up (slope +ve)
  - Generate a 0 if staircase goes down (slope -ve)
- Transmit this sequence of 1,0 data (1-bit per sample)
- Receiver uses this bit stream to reconstruct the staircase waveform and approximate the original analogue waveform

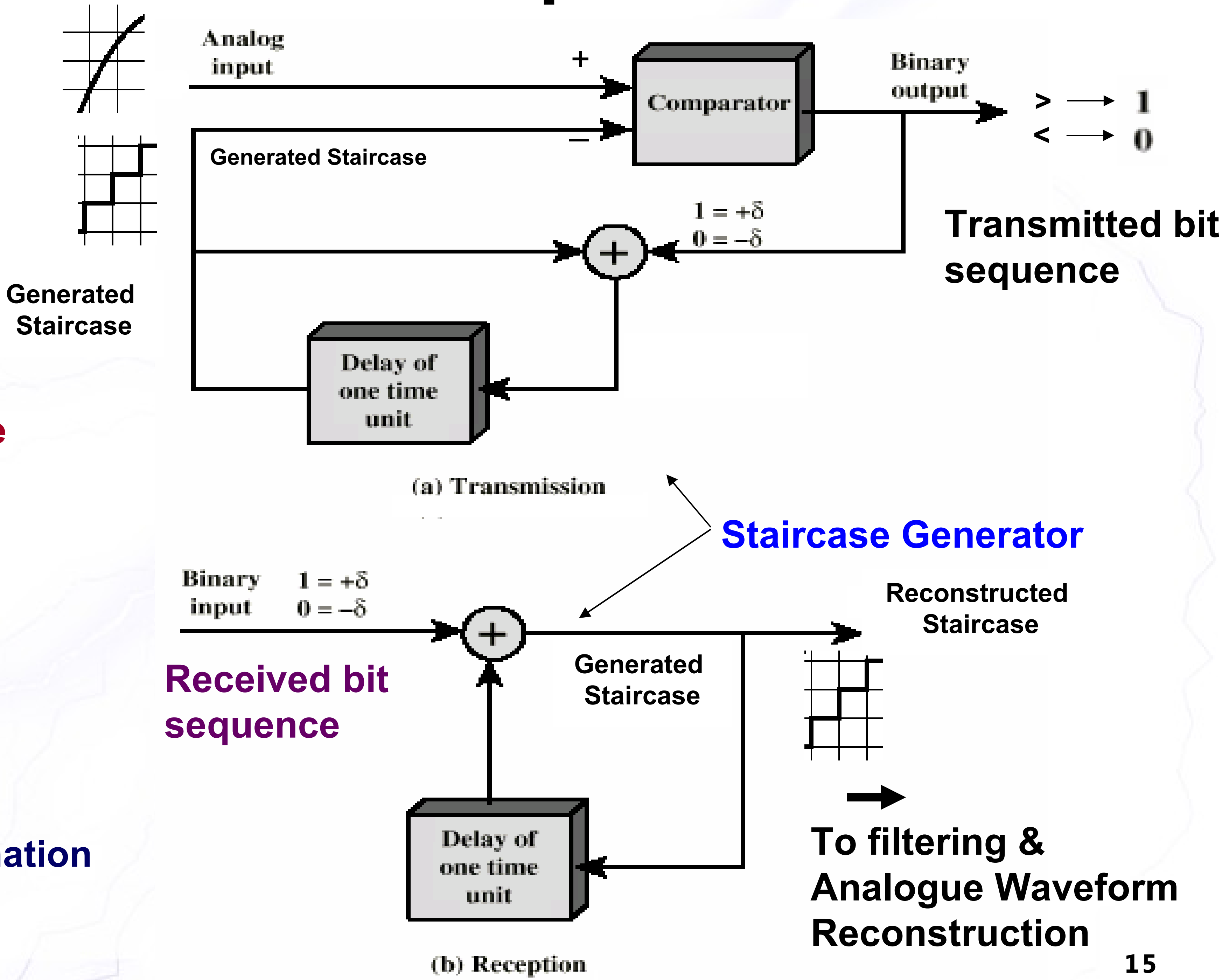
Delta Modulation - example



Delta Modulation - Implementation

- At mid-sampling interval, compare the analogue input to current value of the approximating staircase function
  - If input *exceeds* staircase function, transmit a 1 and increment staircase by δ   for the next sample
  - Otherwise generate a 0 and decrement staircase by δ for the next sample
- Output of the DM is a binary bit sequence to be used for generating the staircase function at RX
  - Reconstruct staircase function at receiving end and smooth by a low pass filter to reconstruct an approximation of the analogue signal

Delta Modulation - Implementation



At Source

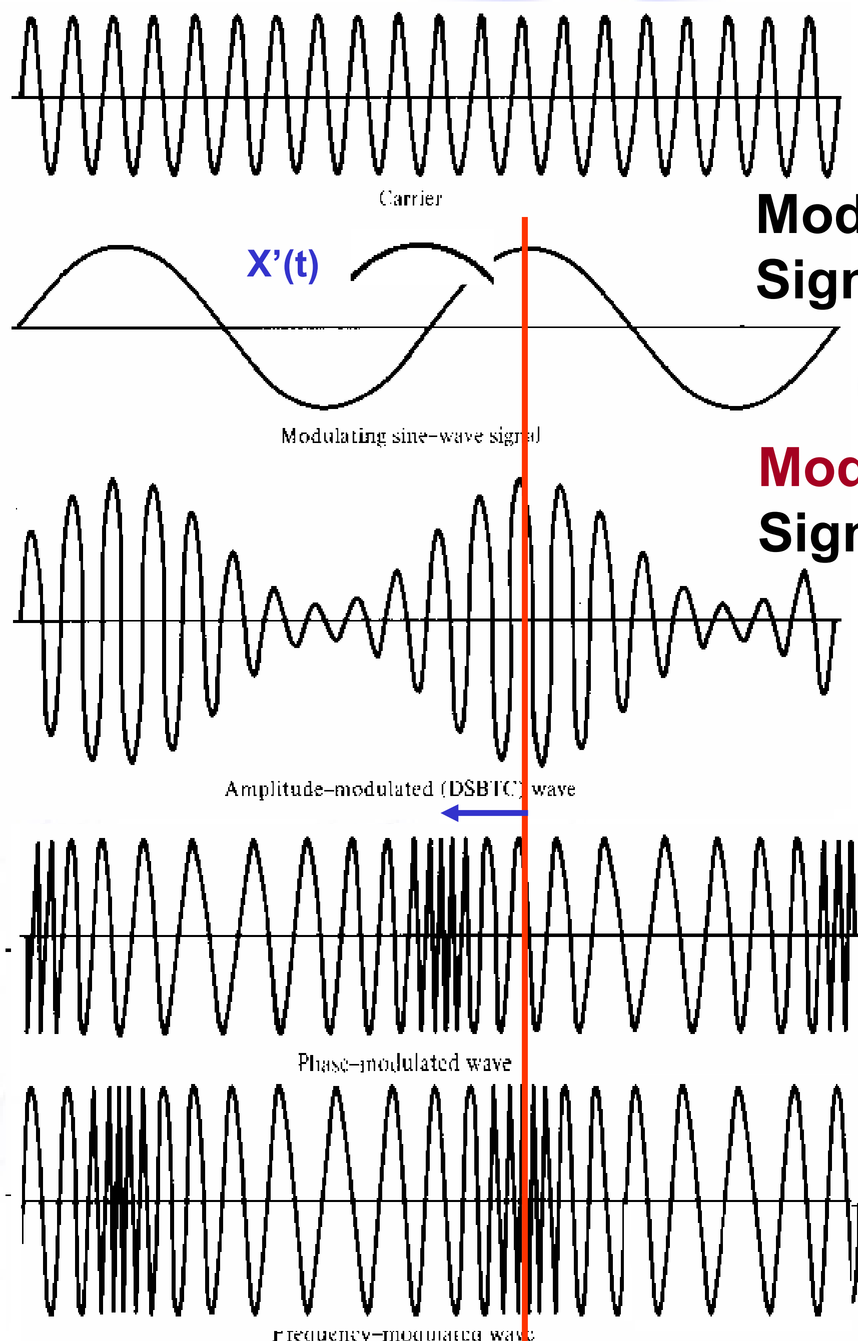
At Destination

Analogue Data, Analogue Signals

- Modulation
  - Combining an input signal *m(t)* and a carrier at frequency f_c to produce signal *s(t)* with bandwidth centered at f_c
- We had to use a form of modulation (shift keying) to represent digital data as analogue signals.
- But why modulate signals that are *already* analogue?
  - Higher frequency may be needed for effective transmission
    - For unguided transmission: impossible to send low frequency baseband signals, e.g. speech, as required antennas would have dimensions in kilometers!
  - Allows implementing frequency division multiplexing (FDM)

Types of Analogue Modulation

Signal to be Transmitted, x(t)



Carrier

Modulating Signal

Modulated Signals

Amplitude Modulation (AM)

$A \propto x(t)$

Angle Modulation:

1. **Phase, PM**

$\phi \propto x(t)$

A sin (ω t)
ϕ (t) = ω t

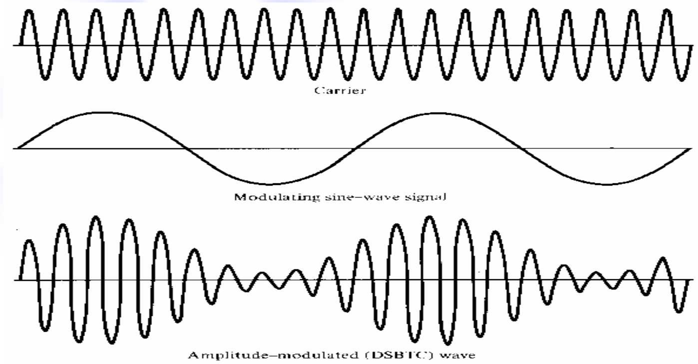
$$\omega = \frac{d\phi(t)}{dt} = \phi'(t)$$

2. **Frequency, FM**

$f \propto x(t)$

Effect of modulation on signal power? **Effect of modulation on signal BW?**

Amplitude Modulation (AM)



- Simplest form of modulation
- $A_c \cos 2\pi f_c t$ is the carrier,
- and x(t)= $A_m \cos 2\pi f_m t$ is the input modulating signal
- Modulated signal expressed as:

$$s(t) = \boxed{[1 + n_a \cos 2\pi f_m t] A_c} \cos 2\pi f_c t$$

Amplitude of modulated wave

Portion of the modulating signal

- **n_a is the modulation index ($0 < n_a \leq 1$):**

$$n_a = \frac{A_m}{A_c}$$

Units of n_a?

- **Added '1' is a DC component to prevent loss of information - there will always be a carrier**
- **Scheme is known as double sideband transmitted carrier (DSBTC)**

Angle Modulation

What parameters can I change to change the angle of the modulated signal?

- Includes:
  - Frequency modulation (FM) and
  - Phase modulation (PM)
- Modulated signal is given by

Total Angle

$$s(t) = A_c \cos[\boxed{2\pi f_c t + \phi(t)}] \qquad \phi(t) = n_p x(t)$$

- Phase modulation (PM): (the direct way)
  - Instantaneous phase is proportional to the modulating signal:
  - n_p is the phase modulation index
- Frequency modulation (FM): (the indirect way)
  - Instantaneous *angular* frequency deviations from ω_c is proportional to the modulating signal,
  $$\phi'(t) = n_f x(t) = \delta\omega(t) = 2\pi\,\delta f(t)$$
  - and we have:
  - So make the *derivative* of ϕ proportional to modulating signal
  - n_f is the frequency modulation index

Angle Modulation

- The total phase angle of s(t) at any instant is $[2\pi f_c t+\phi(t)]$
- Instantaneous phase deviation from that of the carrier is $\phi(t)$
- Phase Modulation (PM):
  - $\phi(t) = n_p x(t)$, instantaneous phase variations are directly proportional to m(t)
- Frequency Modulation (FM):
  - Instantaneous angular frequency, $\omega_i(t)$, can be defined as the rate of change of total phase
  - So, for the modulated signal, s(t)

$$\omega_i(t) = 2\pi f_i(t)$$

$$= \frac{d}{dt}[2\pi f_c t + \phi(t)] = 2\pi f_c + \phi'(t)$$

$$\therefore f_i(t) = f_c + \frac{1}{2\pi}\phi'(t)$$

Phase Modulation (PM)- Example

- Derive an expression for a phase-modulated signal s(t) and its instantaneous frequency given: $A_c = 5V$, and the modulating signal

  $x(t) = 3 \sin 2\pi f_m t$

- We know that s(t):

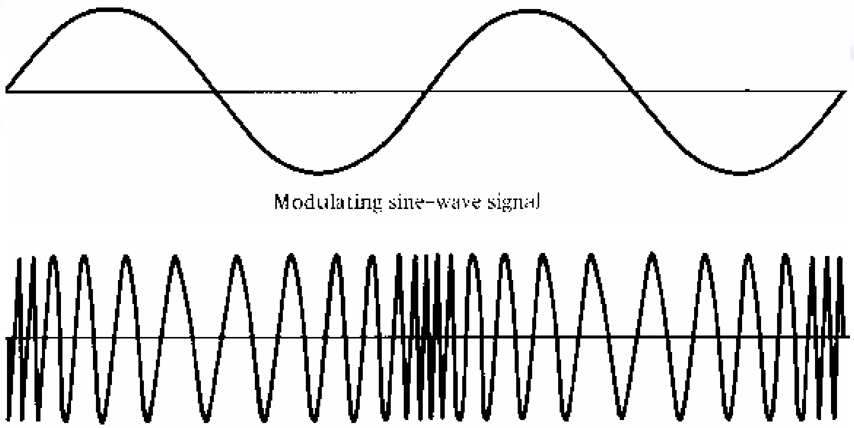
$$s(t) = A_c \cos[2\pi f_c t + \phi(t)]$$

- For PM, $\phi(t)$ is given by:

$$\phi(t) = n_p x(t)$$

n_p is Radians/Volt

- Then s(t) is:



$$s(t) = 5\cos[2\pi f_c t + n_p 3 \sin 2\pi f_m t]$$

- Instantaneous frequency of s(t) is:

Peak frequency deviation for the PM signal

$$f_i(t) = f_c + \frac{3n_p(2\pi f_m)}{2\pi} \cos 2\pi f_m t = f_c + 3n_p f_m \cos 2\pi f_m t$$

$$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}[\text{total phase}]$$

Note: Frequency variations in s(t) *phase-lead* x(t) amplitude variations by 90°

Frequency Modulation: FM

$$f_i = f_c + \frac{1}{2\pi}\phi'(t)$$

$$and\ \phi'(t) = n_f x(t)$$

$$and\ x(t) = A_m \sin(2\pi f_m t)$$

- From equations opposite,

  Peak frequency deviation Δ F is given by:

$$\Delta F = \frac{1}{2\pi} n_f A_m \ \ \text{Hz}$$

  - Where A_m is the peak value of the modulating signal x(t)
- An increase in the amplitude A_m of x(t):

  increases Δ F → increases bandwidth requirement B_T
- But average power level of the FM modulated signal is fixed at $A_C^2/2$, (does not increase with A_m)
- i.e. in Frequency Modulation (angle modulation in general), A_m affects the BW but not the power budget
- While in Amplitude Modulation, A_m affects the power budget but not the bandwidth

Frequency Modulation - Example

- Derive an expression for a frequency-modulated signal s(t) with A_c= 5V, given the modulating signal

$$x(t) = 3\sin 2\pi f_m t$$

- The FM modulated signal s(t) is:

$$s(t) = A_c \cos[2\pi f_c t + \phi(t)]$$

- For FM, $\phi'(t)$ is given by:

$$\phi'(t) = n_f x(t)$$

n_f is (Radians/s)/Volt

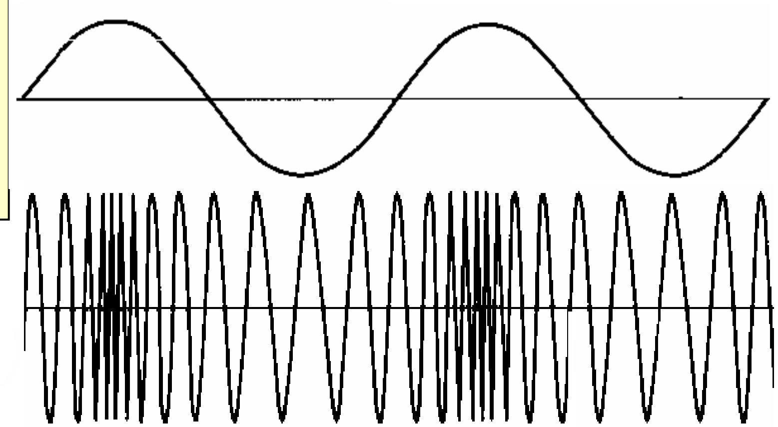
- Then $\phi(t)$ is:

$$\phi(t) = \int \phi'(t)dt = \int n_f 3\sin 2\pi f_m t \; dt = \frac{-3n_f}{2\pi f_m}\cos 2\pi f_m t$$

- We have: $\Delta F = \frac{3}{2\pi} n_f$ Hz

- Substituting for Δ F we get:

But frequency varies as ϕ ', i.e. as sin not as – cos !!



$$s(t) = 5\cos[2\pi f_c t - \frac{\Delta F}{f_m}\cos 2\pi f_m t]$$

Bandwidth Requirement

- All AM, FM, and PM result in a modulated signal whose bandwidth is centered around f_c
- Let B be the bandwidth of the modulating signal (0-B Hz)
- AM gives only sums & differences of frequencies with f_c, and we have: $B_T = 2B$ for DSB systems
- Angle modulation includes a term of the form cos(…+cos()) which is a nonlinear term producing a wide range of frequencies f_c+f_m, f_c+2f_m, … (the Bessel function)
- i.e. Theoretically, an infinite bandwidth is required to transmit an FM or PM signal